# Economics 413: Economic Forecast and Analysis Department of Economics, Finance and Legal Studies University of Alabama 

## Problem Set \#1 - Answers (Linear Regression)

1. (a) Neither figure appears to have a constant mean or variance.


(b) They appear to move together. In general, the one-year rate is greater than the three-month rate, as expected.

(c)

$$
T \widehat{B 1 y r_{t}}=\underset{(0.067)}{0.698}+\underset{(0.010)}{0.917 T B 3 m o_{t}}
$$

(d) The positive coefficient near one tells us that long-run and short-run rates move in the same direction. It also tells us that a one-unit increase in the short term rate is followed by a one-unit increase in the long term rate.
(e) $t=\frac{0.916-1}{0.010}=-89.30 \Rightarrow$ we reject the null that $\beta=1$
(f) We not observe any particular pattern.

(g) The test-statistic for White's test of heteroskedasticity is 31.37 with a $p$-value equal to 0.000 . Therefore we reject the null of homoskedasticity.
(h)

$$
T \widehat{B 1 y} r_{t}=\underset{(0.136)}{0.698}+\underset{(0.025)}{0.917 T B 3 m o_{t}}
$$

(i) We see that both standard errors increase and that the coefficients do not change (as is the case for any data set). Recall that only the standard errors change when using robust standard errors after OLS regression.
(j) Adding the new variable via the equation $\mathrm{d}=(\mathrm{TB} 3 \mathrm{mo}>10)$, we can run the regression

$$
\widehat{T B 1 y} r_{t}=\underset{(0.081)}{0.556}+\underset{(0.014)}{0.945 T B 3 m o_{t}-\underset{(0.152)}{0.446} D_{t}}
$$

(k) $t=\frac{-0.445-0}{0.152}=-2.940$ and the $p$-value is $0.004 \Rightarrow$ we reject the null that the dummy variable is irrelevant
(l) The fit here (in terms of $R^{2}$ ) is raised to 0.981 from 0.979 . We also see decreases in $A I C$ and $S C$.

# Economics 413: Economic Forecast and Analysis Department of Economics, Finance and Legal Studies University of Alabama 

Problem Set \#2 - Answers (ARMA models)

1. (a) The mean does not appear to be constant over time, but the variance appears to be constant.

(b) There is geometric decay in the ACF and a single significant spike in the PACF.

(c)

$$
\begin{aligned}
\widehat{Y 1} & =-0.481+0.753 Y 1_{t-1} \\
\widehat{Y 1}_{t} & =-0.463+0.688 Y 1_{t-1}+0.087 Y 1_{t-2} \\
\widehat{Y 1}_{t} & =-0.460+0.805 Y 1_{t-1}-0.122 \varepsilon_{t-1} \\
\widehat{Y 1}_{t} & =-0.378+0.919 Y 1_{t-1}-0.245 \varepsilon_{t-1}-0.092 \varepsilon_{t-2}-0.084 \varepsilon_{t-3}-0.087 \varepsilon_{t-4} \\
\widehat{Y 1} & =-0.385+1.460 Y 1_{t-1}-0.497 Y 1_{t-2}-0.780 \varepsilon_{t-1}
\end{aligned}
$$

(d)

$$
\begin{aligned}
& \widehat{Y 1_{t}}=0.702 Y 1_{t-1}+0.104 Y 1_{t-2} \\
& \widehat{Y 1_{t}}=0.838 Y 1_{t-1}-0.146 \varepsilon_{t-1}
\end{aligned}
$$

(e) Each goodness-of-fit measure points to the $\operatorname{AR}(1)$ model: $\bar{R}^{2}=0.562, A I C=$ 273.279, and $S C=281.094$.
(f) No. The data is a simulated series from an $\operatorname{AR}(1)$ process. We should be surprised if the correctly specified model isn't preferred.
(g) The residuals appear to be white noise.

2. (a) The series trends upwards over time and is definitely not stationary.

(b) There is a very slow decay in the ACF and a single spike near 1.00 in the PACF.

(c) There appears to be a change in the mean from 1975 to 1985.

(d) The ACF now shows geometric decay.

(e) Again, we find a large change in the mean from 1975 to 1985. However, this series appears to be less volatile.

(f) There is geometric decay in the ACF and 3 or 5 seemingly significant spikes in the PACF at lags 1,2 and 5.

(g) Referring to $\log \left(C P I N S A_{t} / C P I N S A_{t-4}\right)$ as $y_{t}$ we have

$$
\begin{aligned}
& \widehat{y_{t}}=0.033+0.978 y_{t-1} \\
& \widehat{y_{t}}=0.037+1.449 y_{t-1}-0.482 y_{t-2} \\
& \widehat{y_{t}}=0.035+1.443 y_{t-1}-0.433 y_{t-2}+0.217 y_{t-3}-0.628 y_{t-4}+0.377 y_{t-5}
\end{aligned}
$$

(h) The $\operatorname{AR}(5)$ has the largest $\bar{R}^{2}=0.972$, and the smallest $A I C=-1510.859$ and $S C=-1488.167$.
(i) Referring to $\log \left(C P I N S A_{t} / C P I N S A_{t-1}\right)$ as $y_{t}$ we have

$$
\widehat{y_{t}}=0.011-0.003 D_{1 t}-0.001 D_{2 t}-0.002 D_{3 t}
$$

(j) We see some jumps in the residuals from 1975 to 1985. They do not appear to be mean zero over time.

(k) The ACF and PACF do not show that the residuals are not white noise. We should go back and try to find a better model.


# Economics 413: Economic Forecast and Analysis Department of Economics, Finance and Legal Studies University of Alabama 

Problem Set \#3 - Answers (Forecasting)

1. (a) Create $s_{t}$ via generating the variable st $=r 10-\mathrm{Tbill}$. The series appears to be more or less stationary.

(b) The ACF shows decay and the PACF has one prominent spike at lag 1 and perhaps several other significant, but smaller, spikes.

(c)

$$
\widehat{s}_{t}=1.359+1.105 s_{t-1}-0.244 s_{t-2}
$$

(d) The ACF and PACF of the residuals have very small, but sometimes significant spikes.

(e)

$$
\begin{aligned}
\hat{s}_{t}= & 1.368+1.173 s_{t-1}-0.464 s_{t-2}+0.381 s_{t-3}-0.331 s_{t-4} \\
& +0.312 s_{t-5}-0.371 s_{t-6}+0.146 s_{t-7}
\end{aligned}
$$

(f) The ACF and PACF still have some significant spikes.

(g) All signs point to the $\mathrm{AR}(7)$ model. It encompasses the $\mathrm{AR}(2)$ model. The $\mathrm{AR}(2)$ model is more parsimonious, but perhaps too parsimonious.
(h) First, we try to estimate the value of $s_{t}$ at time period 2005:Q4, 0.66

$$
\begin{aligned}
& \widehat{e}_{t+1}=y_{t+1}-\widehat{y}_{t+1 \mid t} \\
& =0.66-\left(1.386+1.096 s_{t}-0.238 s_{t-1}\right) \\
& =0.66-(1.386+1.096 \times 0.84-0.238 \times 1.29) \\
& =-1.340 \\
& \widehat{e}_{t+1}=y_{t+1}-\widehat{y}_{t+1 \mid t} \\
& =0.66-\binom{1.383+1.177 s_{t-1}-0.480 s_{t-2}+0.396 s_{t-3}-0.345 s_{t-4}}{+0.328 s_{t-5}-0.387 s_{t-6}+0.163_{t-7}} \\
& =0.66-\binom{1.383+1.177 \times 0.84-0.480 \times 1.29+0.396 \times 1.77-0.345 \times 2.17}{+0.328 \times 2.81-0.387 \times 3.52+0.163 \times 3.52} \\
& =-1.178
\end{aligned}
$$

This result should be expected as we have previously argued that the $\mathrm{AR}(7)$ model is a better model than the $\operatorname{AR}(2)$ model for this particular data set.
(i) The Root Mean Squared Error are 0.798 and 0.924 for the $\operatorname{AR}(2)$ and $\operatorname{AR}(7)$ models, respectively. We get a flip here. It looks like the more parsimonious model is better for longer term forecasts. Perhaps there is too much variability when estimating the additional terms.


## Economics 413: Economic Forecast and Analysis Department of Economics, Finance and Legal Studies University of Alabama

Problem Set \#4 - Answers (Univariate Nonstationary Time Series)

1. (a) There is an upward sloping trend. The series is definitely nonstationary.

(b)

$$
\widehat{G D P}{ }_{t}=2223.94+38.510 t-0.171 t^{2}+0.001 t^{3}
$$

(c) There is a large spike near 1.00 (0.925) in the PACF and decay in the ACF.

(d) Testing for no lags and no constant, our test statistic is 13.363 and our $p$-value is equal to 1.000 . Hence, we fail to reject the null of a unit root.
(e) The series appears to be stationary.

(f)

$$
d \widehat{l r g} d p_{t}=0.008+0.262 d l r g d p_{t-1}+0.150 d l r g d p_{t-2}
$$

(g) The ACF and PACF lags for the residuals are very small and the series appears to be white noise.

(h) Testing for a single lag and no constant, our test statistic is -6.085 and our $p$-value is 0.000 . Hence, we reject the null of a unit root.

# Economics 413: Economic Forecast and Analysis Department of Economics, Finance and Legal Studies University of Alabama 

## Problem Set \#5 - Answers (Multivariate Nonstationary Time Series)

1. (a) indpro and $M 1 N S A$ have an upward trend. Each appear to be nonstationary. urate appears to be stationary.



(b) Testing for no lags and no constants, our test statistic is 7.413 and our $p$-value is 1.000. Hence, we fail to reject the null of a unit root.
(c) Testing for no lags and no constantst, our test statistic is 7.300 and our $p$-value is 1.000 . Hence, we fail to reject the null of a unit root.
(d)

$$
\widehat{i n d p r} o_{t}=28.537+0.056 M 1 N S A_{t}
$$

(e) They have a single tall spike in the PACF and a relatively slow decay in the ACF. They may be nonstationary.

(f) Our test statistic is -1.259 and our $p$-value is 0.1912 . Hence, we fail to reject the null of a unit root.
(g) The two series move with one another, but do not necessarily cause one another. This likely is a spurious regression.

2. (a) Each series appears to be nonstationary.

(b) Testing for no lags and no constants, for $\log \left(e x \_c a\right)$ has a test statistic equal to 0.019 and a $p$-value equal to 0.687 . $\log \left(p \_u s\right)$ has a test statistic equal to 7.592 and a $p$-value equal to $1.000 . \log \left(p_{-} c a\right)$ has a test statistic equal to 8.061 and a $p$-value equal to 1.000 . In each case, we fail to reject the null of a unit root.
(c)

$$
\log \left(\widehat{e x_{-}} c a_{t}\right)=4.766+1.376 \log \left(p_{\_} c a_{t}\right)-1.405 \log \left(p_{\_} u s_{t}\right)
$$

(d) The coefficients are fairly equal in magnitude, and have different signs. Absolute PPP also requires that each coefficient is near 1.00. Relative PPP allows the coefficients to be equal in magnitude and opposite signs, but this requires us to regress on growth rates and not logs. In short, it appears that PPP does not hold.
(e) Since the residuals from the equilibrium regression appear to contain a unit root, shocks to the real exchange rate never decay. Hence, long-run PPP fails.


