

Economics 513: Economic Forecast and Analysis
 Department of Economics, Finance and Legal Studies
 University of Alabama

Problem Set #2 – Answers (ARMA models)

1. (a) The mean does not appear to be constant over time, but the variance appears to be constant.
- (b) There is geometric decay in the ACF and a single significant spike in the PACF.
- (c)

$$\widehat{Y}1_t = -0.559 + 0.754Y1_{t-1}$$

$$\widehat{Y}1_t = -0.522 + 0.694Y1_{t-1} + 0.087Y1_{t-2}$$

$$\widehat{Y}1_t = -0.549 + 0.805Y1_{t-1} - 0.116\varepsilon_{t-1}$$

$$\widehat{Y}1_t = -0.520 + 0.919Y1_{t-1} - 0.245\varepsilon_{t-1} - 0.089\varepsilon_{t-2} - 0.084\varepsilon_{t-3} - 0.090\varepsilon_{t-4}$$

$$\widehat{Y}1_t = -0.539 - 0.031Y1_{t-1} + 0.617Y1_{t-2} + 0.762\varepsilon_{t-1}$$

(d)

$$\widehat{Y}1_t = 0.710Y1_{t-1} + 0.105Y1_{t-2}$$

$$\widehat{Y}1_t = 0.846Y1_{t-1} - 0.147\varepsilon_{t-1}$$

- (e) Each goodness-of-fit measure points to the AR(1) model: $\overline{R}^2 = 0.557$, $AIC = 2.708$, and $SC = 2.761$.
 - (f) No. The data is a simulated series from an AR(1) process. We should be surprised if the correctly specified model isn't preferred.
 - (g) The residuals appear to be white noise.
2. (a) The series trends upwards over time and is definitely not stationary.
 - (b) There is a very slow decay in the ACF and a single spike near 1.00 in the PACF.
 - (c) There appears to be a change in the mean from 1975 to 1985.
 - (d) The ACF now shows geometric decay.
 - (e) Again, we find a large change in the mean from 1975 to 1985. However, this series appears to be less volatile.
 - (f) There is geometric decay in the ACF and 3 or 5 seemingly significant spikes in the PACF at lags 1, 2 and 5.
 - (g) Referring to $\log(CPINSAt/CPINSA_{t-4})$ as y_t we have

$$\widehat{y}_t = 0.044 + 0.974y_{t-1}$$

$$\widehat{y}_t = 0.042 + 1.448y_{t-1} - 0.484y_{t-2}$$

$$\widehat{y}_t = 0.043 + 1.443y_{t-1} - 0.434y_{t-2} + 0.218y_{t-3} - 0.635y_{t-4} + 0.381y_{t-5}$$

- (h) The AR(5) has the largest $\overline{R}^2 = 0.970$, and the smallest $AIC = -8.014$ and $SC = -7.909$.
- (i) Referring to $\log(CPINS A_t)$ as y_t we have

$$\hat{y}_t = 0.011 - 0.003D_{1t} - 0.001D_{2t} - 0.002D_{3t}$$

- (j) We see some jumps in the residuals from 1975 to 1985. They do not appear to be mean zero over time.
- (k) The ACF and PACF do not show that the residuals are white noise. We should go back and try to find a better model.

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Problem Set #3 – Answers (Forecasting)

1. (a) The series appears to be more or less stationary.
- (b) The ACF shows decay and the PACF has one prominent spike at lag 1 and perhaps several other significant, but smaller, spikes.
- (c) $\hat{s}_t = 1.388 + 1.109s_{t-1} - 0.245s_{t-2}$
- (d) The ACF and PACF of the residuals have very small, but sometimes significant spikes.
- (e)

$$\begin{aligned}\hat{s}_t &= 1.389 + 1.179s_{t-1} - 0.471s_{t-2} + 0.392s_{t-3} - 0.345s_{t-4} \\ &\quad + 0.324s_{t-5} - 0.383s_{t-6} + 0.152s_{t-7}\end{aligned}$$

- (f) The ACF and PACF of the residuals have very small and insignificant spikes.
- (g) All signs point to the AR(7) model. It encompasses the AR(2) model. The AR(2) model is more parsimonious, but perhaps too parsimonious.
- (h) First, we try to estimate the value of s at time period 2005:Q4, 0.66

$$\begin{aligned}\hat{e}_{t+1} &= y_{t+1} - \hat{y}_{t+1|t} \\ &= 0.66 - (1.388 + 1.109s_t - 0.245s_{t-1}) \\ &= 0.66 - (1.388 + 1.109 \times 0.84 - 0.245 \times 1.29) \\ &= -1.344\end{aligned}$$

$$\begin{aligned}\hat{e}_{t+1} &= y_{t+1} - \hat{y}_{t+1|t} \\ &= 0.66 - \left(\begin{array}{l} 1.389 + 1.179s_t - 0.471s_{t-1} + 0.392s_{t-2} - 0.345s_{t-3} \\ + 0.324s_{t-4} - 0.383s_{t-5} + 0.152s_{t-6} \end{array} \right) \\ &= 0.66 - \left(\begin{array}{l} 1.389 + 1.179 \times 0.84 - 0.471 \times 1.29 + 0.392 \times 1.77 - 0.345 \times 2.17 \\ + 0.324 \times 2.81 - 0.383 \times 3.52 + 0.152 \times 3.52 \end{array} \right) \\ &= -1.090\end{aligned}$$

This result should be expected as we have previously argued that the AR(7) model is a better model than the AR(2) model for this particular data set.

- (i) These values are 1.729 and 1.610 for the AR(2) and AR(7) models, respectively. Again, as expected, the AR(7) model performs better.

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Problem Set #4 – Answers (Univariate Nonstationary Time Series)

1. (a) There is an upward sloping trend. The series is definitely nonstationary.
- (b) $\widehat{GDP}_t = 2262.277 + 38.173t - 0.1667t^2 + 0.001t^3$
- (c) There is a large spike near 1.00 (0.925) in the PACF and decay in the ACF.
- (d) Our test statistic is 2.669 and our p -value is equal to 1.000. Hence, we fail to reject the null of a unit root.
- (e) The series appears to be stationary.
- (f) $\widehat{dlrgdp}_t = 0.008 + 0.256dlrgdp_{t-1} + 0.150dlrgdp_{t-2}$
- (g) The ACF and PACF lags for the residuals are very small and the series appears to be white noise.
- (h) Our test statistic is -6.536 and our p -value is 0.000. Hence, we reject the null of a unit root.
- (i) Our F -statistic is 1.222 and our p -value is 0.303. Hence, we fail to reject the null of a break point in 1973:Q1.
- (j) $\widehat{dlrgdp}_t = 0.010 + 0.242dlrgdp_{t-1} + 0.138dlrgdp_{t-2} - 0.003D_t$
- (k) The ACF and PACF lags for the residuals are very small and the series appears to be white noise.

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Problem Set #5 – Answers (Multivariate Nonstationary Time Series)

1. (a) *indpro* and *M1NSA* have an upward trend. Each appear to be nonstationary
(b) Our test statistic is 0.175 and our p -value is 0.970. Hence, we fail to reject the null of a unit root.
(c) Our test statistic is 1.138 and our p -value is 0.998. Hence, we fail to reject the null of a unit root.
(d) $\widehat{indpro}_t = 28.537 + 0.056M1NSA_t$
(e) They have a single tall spike in the PACF and a relatively slow decay in the ACF. They may be nonstationary.
(f) Our test statistic is -2.097 and our p -value is 0.246. Hence, we fail to reject the null of a unit root.
(g) The two series move with one another, but do not necessarily cause one another. This likely is a spurious regression.
2. (a) Each series appears to be nonstationary.
(b) $\log(ex_ca)$ has a test statistic equal to -1.456 and a p -value equal to 0.553. Hence, we fail to reject the null of a unit root. $\log(p_ca)$ has a test statistic equal to -5.208 and a p -value equal to 0.000. Hence, we reject the null of a unit root. $\log(p_us)$ has a test statistic equal to -1.809 and a p -value equal to 0.3752. Hence, we fail to reject the null of a unit root (at the 5% level).
(c) $\log(\widehat{ex_ca}_t) = 4.713 + 1.339 \log(p_ca_t) - 1.358 \log(p_us_t)$
(d) The coefficients are fairly equal in magnitude, and have different signs. Absolute PPP also requires that each coefficient is near 1.00. Relative PPP allows the coefficients to be equal in magnitude and opposite signs, but this requires us to regress on growth rates and not logs. In short, it appears that PPP does not hold.
(e) Since the residuals from the equilibrium regression appear to contain a unit root, shocks to the real exchange rate never decay. Hence, long-run PPP fails.