$V\left(\widehat{\beta}\right)=\frac{\sigma^2}{\sum\limits_{i=1}^{n}(x_i-\overline{x})^2}$

Proof:

$$\begin{aligned}
\widehat{\beta} &= \frac{\sum\limits_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}{\sum\limits_{i=1}^{n}(x_i-\overline{x})^2} = \frac{\sum\limits_{i=1}^{n}(x_i-\overline{x})y_i}{\sum\limits_{i=1}^{n}(x_i-\overline{x})^2} = \frac{\sum\limits_{i=1}^{n}(x_i-\overline{x})(\alpha+\beta x_i+u_i)}{\sum\limits_{i=1}^{n}(x_i-\overline{x})^2} \\
&= \alpha\frac{\sum\limits_{i=1}^{n}(x_i-\overline{x})}{\sum\limits_{i=1}^{n}(x_i-\overline{x})^2} + \beta\frac{\sum\limits_{i=1}^{n}(x_i-\overline{x})x_i}{\sum\limits_{i=1}^{n}(x_i-\overline{x})^2} + \frac{\sum\limits_{i=1}^{n}(x_i-\overline{x})u_i}{\sum\limits_{i=1}^{n}(x_i-\overline{x})^2} \\
&= 0+\beta+\frac{\sum\limits_{i=1}^{n}(x_i-\overline{x})u_i}{\sum\limits_{i=1}^{n}(x_i-\overline{x})^2}
\end{aligned}$$

$$\begin{aligned}
V\left(\widehat{\beta}\right) &= V\left(\beta+\frac{\sum\limits_{i=1}^{n}(x_i-\overline{x})u_i}{\sum\limits_{i=1}^{n}(x_i-\overline{x})^2}\right) \\
&= \frac{1}{\left(\sum\limits_{i=1}^{n}\left(x_i-\overline{x}\right)^2\right)^2}\sum_{i=1}^{n}V\left(\left(x_i-\overline{x}\right)u_i\right) \\
&= \frac{1}{\left(\sum\limits_{i=1}^{n}\left(x_i-\overline{x}\right)^2\right)^2}\sum_{i=1}^{n}\left(x_i-\overline{x}\right)^2 V\left(u_i\right) \\
&= \frac{\sigma^2}{\left(\sum\limits_{i=1}^{n}\left(x_i-\overline{x}\right)^2\right)^2}\sum_{i=1}^{n}\left(x_i-\overline{x}\right)^2 \\
&= \frac{\sigma^2}{\sum\limits_{i=1}^{n}\left(x_i-\overline{x}\right)^2}
\end{aligned}$$

$V\left(\widehat{\alpha}\right)=\frac{\frac{\sigma^2}{n}\sum\limits_{i=1}^{n}x_i^2}{\sum\limits_{i=1}^{n}(x_i-\overline{x})^2}$

Proof:

$$\begin{aligned}
\widehat{\alpha} &= \overline{y}-\widehat{\beta}\overline{x}=\left(\alpha+\beta\overline{x}+\overline{u}\right)-\widehat{\beta}\overline{x} \\
&= \alpha+\left(\beta-\widehat{\beta}\right)\overline{x}+\overline{u}=\alpha+\left(\beta-\widehat{\beta}\right)\overline{x}+\frac{1}{n}\sum_{i=1}^{n}u_i
\end{aligned}$$

$$\begin{aligned}
V\left(\widehat{\alpha}\right) &= V\left(\alpha+\left(\beta-\widehat{\beta}\right)\overline{x}+\frac{1}{n}\sum_{i=1}^{n}u_i\right) \\
&= V\left(\alpha\right)+V\left(\left(\beta-\widehat{\beta}\right)\overline{x}\right)+V\left(\frac{1}{n}\sum_{i=1}^{n}u_i\right) \\
&= 0+\overline{x}^2V\left(\widehat{\beta}\right)+\frac{1}{n^2}\sum_{i=1}^{n}V\left(u_i\right) \\
&= \overline{x}^2\frac{\sigma^2}{\sum\limits_{i=1}^{n}\left(x_i-\overline{x}\right)^2}+\frac{1}{n^2}n\sigma^2=\frac{\sigma^2}{n}+\overline{x}^2\frac{\sigma^2}{\sum\limits_{i=1}^{n}\left(x_i-\overline{x}\right)^2} \\
&= \frac{\sigma^2\sum\limits_{i=1}^{n}\left(x_i-\overline{x}\right)^2+n\overline{x}^2\sigma^2}{n\sum\limits_{i=1}^{n}\left(x_i-\overline{x}\right)^2}=\frac{\frac{\sigma^2}{n}\sum\limits_{i=1}^{n}\left(x_i-\overline{x}\right)^2+\overline{x}^2\sigma^2}{\sum\limits_{i=1}^{n}\left(x_i-\overline{x}\right)^2} \\
&= \frac{\frac{\sigma^2}{n}\sum\limits_{i=1}^{n}\left(x_i^2-2x_i\overline{x}+\overline{x}^2\right)+\overline{x}^2\sigma^2}{\sum\limits_{i=1}^{n}\left(x_i-\overline{x}\right)^2} \\
&= \frac{\frac{\sigma^2}{n}\left(\sum\limits_{i=1}^{n}x_i^2-2\overline{x}\sum\limits_{i=1}^{n}x_i+n\overline{x}^2\right)+\overline{x}^2\sigma^2}{\sum\limits_{i=1}^{n}\left(x_i-\overline{x}\right)^2} \\
&= \frac{\frac{\sigma^2}{n}\left(\sum\limits_{i=1}^{n}x_i^2-2n\overline{x}^2+n\overline{x}^2\right)+\overline{x}^2\sigma^2}{\sum\limits_{i=1}^{n}\left(x_i-\overline{x}\right)^2} \\
&= \frac{\frac{\sigma^2}{n}\sum\limits_{i=1}^{n}x_i^2-2\sigma^2\overline{x}^2+\sigma^2\overline{x}^2+\sigma^2\overline{x}^2}{\sum\limits_{i=1}^{n}\left(x_i-\overline{x}\right)^2}=\frac{\frac{\sigma^2}{n}\sum\limits_{i=1}^{n}x_i^2}{\sum\limits_{i=1}^{n}\left(x_i-\overline{x}\right)^2}
\end{aligned}$$