

# Economics 673: Applied Nonparametric Econometrics

Department of Economics, Finance and Legal Studies

University of Alabama

Fall 2020

Midterm

The exam consists of three questions on three pages. Each question is of equal value.

1. Suppose that instead of the density itself,  $\hat{f}(x) = \frac{1}{nh} \sum_{i=1}^n k\left(\frac{x_i - x}{h}\right)$ , we are interested in its derivative,  $\hat{f}'(x) = \partial \hat{f}(x) / \partial x$ . Derive the estimator  $\hat{f}'(x)$  assuming the kernel is Gaussian. Why can we not use a uniform kernel here?
2. Consider the case of a conditional density of a scalar  $y$  on a  $q$ -vector  $x$  (i.e.,  $f(y|x) = f(y, x) / f(x)$ ). Give the kernel density estimator  $\hat{f}(y|x)$ . Now, suppose the  $q$ th element of  $x$  is not a relevant predictor of  $y$ . As  $n \rightarrow \infty$ , what will happen to the vector of bandwidths for the  $q$ -vector  $x$  and  $y$  when they are estimated via least-squares cross-validation? Show this result on your conditional density estimator  $\hat{f}(y|x)$ .
3. Below is the kernel estimated density (Gaussian kernel with the Silverman (1986) rule-of-thumb bandwidth) of average yearly snowfall (in inches) in Buffalo, New York. Considering the figure, what do you expect to be the outcome of a test for (a) that the correctly specified density is Gaussian and (b) the density is symmetric. In each of these settings, explain how you would impose the null hypothesis in a bootstrap procedure.

