Economics 671: Applied Econometrics
Department of Economics, Finance and Legal Studies
University of Alabama
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Midterm Answers

1. (a)

$$\begin{aligned} f_Y(y) &= \exp\left\{\ln\left[\frac{y^{a-1}\exp\left(-\frac{ya}{\mu}\right)}{\Gamma(a)\left(\frac{\mu}{a}\right)^a}\right]\right\} \\ &= \exp\left[(a-1)\ln y - \frac{ya}{\mu} - \ln\Gamma(a) - a\ln\frac{\mu}{a}\right] \\ &= \exp\left\{[-a\ln\mu] + [-a\ln a + (a-1)\ln y - \ln\Gamma(a)] + \left[-\frac{a}{\mu}\right]y\right\} \end{aligned}$$

(b)

$$\begin{aligned} \ln L(\theta, y) &= \sum_{i=1}^{n} \ln f_Y(y_i) \\ &= \sum_{i=1}^{n} \ln\left\{\exp\left\{\begin{array}{c} [-a\ln\exp(x_i\beta)] + [-a\ln a\,(a-1)\ln y_i - \ln\Gamma(a)] \\ + \left[-\frac{a}{\exp(x_i\beta)}\right] y_i \end{array}\right\}\right\} \\ &= \sum_{i=1}^{n}\left\{[-a(x_i\beta)] + [-a\ln a + (a-1)\ln y_i - \ln\Gamma(a)] + \left[-\frac{a}{\exp(x_i\beta)}\right] y_i\right\} \end{aligned}$$

which is equivalent to maximizing

$$\sum_{i=1}^{n} -a(x_i\beta) - \frac{a}{\exp(x_i\beta)} y_i$$

(c)

$$\frac{\partial \ln L(\theta, y)}{\partial \beta} = \sum_{i=1}^{n} -a x_i + \frac{a x_i y_i}{[\exp(x_i\beta)]}$$

(d)

$$\frac{\partial^2 \ln L(\theta, y)}{\partial \beta^2} = \sum_{i=1}^{n} -\frac{a x_i x_i y_i}{[\exp(x_i\beta)]}$$

(e) The second derivative is negative semi-definitive and thus we have a guarantee that the iterative outcome will be unique.

2. (a) The objective function is given as

$$\begin{aligned} Q_n(\theta) &= \left[\frac{1}{n}\sum_{i=1}^{n} u_i' z_i\right] W_n \left[\frac{1}{n}\sum_{i=1}^{n} z_i' u_i\right] \\ &= \left[\frac{1}{n}\sum_{i=1}^{n} (y_i - x_i\beta)' z_i\right] W_n \left[\frac{1}{n}\sum_{i=1}^{n} z_i' (y_i - x_i\beta)\right] \end{aligned}$$

where W_n is a symmetric positive defninite weighting matrix.

(b) The first order conditons are

$$\frac{\partial Q_n(\theta)}{\partial \beta} = \left[\frac{1}{n}\sum_{i=1}^{n} x_i' z_i\right] W_n \left[\frac{1}{n}\sum_{i=1}^{n} z_i' y_i\right] - \left[\frac{1}{n}\sum_{i=1}^{n} x_i' z_i\right] W_n \left[\frac{1}{n}\sum_{i=1}^{n} z_i' x_i\right] \widehat{\beta}$$

(c) Setting the FOC equal to zero an solving for $\widehat{\beta}$ gives

$$\widehat{\beta}_{GMM} = \left\{\left[\frac{1}{n}\sum_{i=1}^{n} x_i' z_i\right] W_n \left[\frac{1}{n}\sum_{i=1}^{n} z_i' x_i\right]\right\}^{-1} \left[\frac{1}{n}\sum_{i=1}^{n} x_i' z_i\right] W_n \left[\frac{1}{n}\sum_{i=1}^{n} x_i' y_i\right]$$

3. (a) T_{ij} is the trade from country i to country j. Y_i is the level of output for country i. Y_j is the level of output for country j. D_{ij} is the measure of distance between countries i and j. ε_{ij} is the standard error term which is assumed to have a mean equal to unity.

   (b) α_0 is often referred to as a technology paramter. A larger value of α_0 makes trade easier. It must be positive. α_1 an elasticity and is assumed to be positive. Higher output for country i should lead to more trade. α_2 an elasticity and is assumed to be positive, but it can be negative (Engel's Law). Higher output from country j should lead to more trade as well. α_3 an elasticity and is assumed to be negative. Further distance should have a negative effect on trade from country i to country j.

   (c)

$$\begin{aligned} E\left(T_{ij}|Y_i, Y_j, D_{ij}\right) &= E\left[\exp\left(\ln \alpha_0 + \alpha_1 \ln Y_i + \alpha_2 \ln Y_j + \alpha_3 \ln D_{ij}\right)\varepsilon_{ij}|Y_i, Y_j, D_{ij}\right] \\ &= \exp\left(\ln \alpha_0 + \alpha_1 \ln Y_i + \alpha_2 \ln Y_j + \alpha_3 \ln D_{ij}\right) \end{aligned}$$

   (d)

$$\begin{aligned} Q_n(\theta) &= \sum_{i=1}^{n}\sum_{j=1}^{n} - \exp\left(\ln \alpha_0 + \alpha_1 \ln Y_i + \alpha_2 \ln Y_j + \alpha_3 \ln D_{ij}\right) \\ &\quad + T_{ij}\left(\ln \alpha_0 + \alpha_1 \ln Y_i + \alpha_2 \ln Y_j + \alpha_3 \ln D_{ij}\right) - \ln T_{ij}! \end{aligned}$$

$$\begin{aligned} \frac{\partial Q_n(\theta)}{\partial \alpha_0} &= \sum_{i=1}^{n}\sum_{j=1}^{n} - \exp\left(\ln \alpha_0 + \alpha_1 \ln Y_i + \alpha_2 \ln Y_j + \alpha_3 \ln D_{ij}\right)\frac{1}{\alpha_0} + \frac{1}{\alpha_0}T_{ij} \\ \frac{\partial Q_n(\theta)}{\partial \alpha_1} &= \sum_{i=1}^{n}\sum_{j=1}^{n} - \exp\left(\ln \alpha_0 + \alpha_1 \ln Y_i + \alpha_2 \ln Y_j + \alpha_3 \ln D_{ij}\right)\ln Y_i + T_{ij}\ln Y_i \\ \frac{\partial Q_n(\theta)}{\partial \alpha_2} &= \sum_{i=1}^{n}\sum_{j=1}^{n} - \exp\left(\ln \alpha_0 + \alpha_1 \ln Y_i + \alpha_2 \ln Y_j + \alpha_3 \ln D_{ij}\right)\ln Y_j + T_{ij}\ln Y_j \\ \frac{\partial Q_n(\theta)}{\partial \alpha_3} &= \sum_{i=1}^{n}\sum_{j=1}^{n} - \exp\left(\ln \alpha_0 + \alpha_1 \ln Y_i + \alpha_2 \ln Y_j + \alpha_3 \ln D_{ij}\right)\ln D_{ij} + T_{ij}\ln D_{ij} \end{aligned}$$