

Economics 670: Econometrics

Department of Economics, Finance and Legal Studies

University of Alabama

Fall 2021

Midterm II

The exam consists of three questions on three pages. Each question is of equal value.

1. Let $\mu_r = E(Y^r)$ be the r th moment of the random variable $Y \sim (0, \sigma^2)$, where we assume that $E(Y^{2r}) < \infty$. With this information:
 - (a) Write down the natural estimator for $\hat{\mu}_r$ of μ_r .
 - (b) Using large sample theory, show that the estimator from part (a) is consistent.
 - (c) Find the asymptotic distribution of $\sqrt{n}(\hat{\mu}_r - \mu_r)$ as the sample size $n \rightarrow \infty$.

2. Consider the model $y = X\beta + e$, for a random sample of $i = 1, 2, \dots, n$ observations with k regressors, where $e|X \sim N(0, \sigma^2)$. For this model,
- (a) Derive the maximum likelihood estimator of σ^2 .
 - (b) Derive the variance of the estimator in part (a).
 - (c) Using large sample theory, show that the estimator from part (a) is consistent.

3. Consider the model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + e$, where $e|x_1, x_2 \sim N(0, \sigma^2)$ where σ^2 is known and x_1 and x_2 are scalars. Suppose we are interested in testing the null $H_0 : \beta_2 = 0$.

- (a) Write down the test statistic for a t -test. What is the asymptotic distribution of this statistic under the null?
- (b) Write down the test statistic for a likelihood-ratio test. What is the asymptotic distribution of this statistic under the null?
- (c) Write down the test statistic for an F -test. What is the asymptotic distribution of this statistic under the null?