Economics 670: Econometrics

Department of Economics, Finance and Legal Studies
University of Alabama
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Midterm II

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider the model $y = X\beta + e$, for a random sample of $i = 1, 2, \ldots, n$ observations and k regressors, where e is normally distributed with mean 0 and variance σ^2. For this model,
   (a) Derive the OLS estimator of β
   (b) Derive the variance of the estimator in part (a)
   (c) Derive the MLE estimator of β
   (d) Derive the variance of the estimator in part (c)

2. Consider the random variable y which has mean zero and variance σ^2. Define the third moment of y by $\mu_3 = E(y^3)$. For a random sample of size n,

   (a) Construct an estimator $\widehat{\mu}_3$ for μ_3.

   (b) Show that $\widehat{\mu}_3$ is an unbiased estimator for μ_3.

   (c) Calculate the variance of $\widehat{\mu}_3$, say $V(\widehat{\mu}_3)$.

3. Consider the R code below. Next to each line of code, briefly comment on what that line of code is doing.

```
## R code for Question 3 - MT2

        rm(list=ls())

        set.seed(123456)

        n <- 100

        x <- rnorm(n,0,1)

        u <- rnorm(n,0,0.1)

        y <- 1 + 0.5*x + u

        x <- as.matrix(x)

        y <- as.matrix(y)

        xm <- mean(x)

        ym <- mean(y)

        ones <- as.matrix(rep(1,n))

        X <- cbind(ones,x)

        b <- solve(t(X)%*%X)%*%(t(X)%*%y)

        u <- y - X%*%b

        yh <- X%*%b

        st <- sumc((y-ym)^2)

        se <- sumc((yh-ym)^2)

        sr <- sumc((y-yh)^2)

        r2 <- 1-sr/st

        s2 <- solve(n-1)*sumc(u^2)

        vb <- sqrt(s2)*solve(t(X)%*%X)

        t <- (b-0)/sqrt(vb[2,2])
```