Economics 670: Econometrics

Department of Economics, Finance and Legal Studies

University of Alabama

Fall 2020

Midterm II

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider the regression model $y = X\beta + e$, where we assume that $E(e|X) = 0$. For each scenario below, show that the OLS estimator of β is asymptotically normal. Be sure to specify the mean and variance of the estimator (or its normalized version).

   (a) Using the assumption $e|X$ is i.i.d. $N(0, \sigma^2 I_n)$

   (b) Using large sample theory (be sure to note assumptions/laws being used)

2. Consider the regression model $y = X\beta + e$, where $e|X$ i.i.d. $N(0, \sigma^2 I_n)$. Suppose we have a consistent, asymptotically normal estimator $\widehat{\beta}$ of β with asymptotic variance V_β whereby β is a scalar and we have a consistent, asymptotically normal estimator s^2 of σ^2. Consider the statistic

$$T = \frac{\widehat{\beta} - \beta}{se\left(\widehat{\beta}\right)},$$

where $se(\widehat{\beta})$ is the standard error of the estimator (i.e., square root of the estimated variance of $\widehat{\beta}$). With this information, answer the following (show your work):

(a) What is the finite sample distribution of this statistic?

(b) What is the asymptotic distribution of this statistic?

3. Consider the R code that we discussed in lecture. Next to each line of code, briefly comment on what that line of code is doing.

```
n <- 100
b <- 1000

alpha <- numeric(b)
beta <- numeric(b)
clt.alpha <- numeric(b)
clt.beta <- numeric(b)

for (j in 1:b){

u <- runif(n,-0.5,0.5)
x <- rnorm(n,0,1)
y <- 1 + x + u

ols.estimates <- lm(y~x)

alpha[j] <- ols.estimates$coef[1]
beta[j] <- ols.estimates$coef[2]

clt.alpha[j] <- sqrt(n)*(alpha[j]-1)
clt.beta[j] <- sqrt(n)*(beta[j]-1)

}

hist(clt.alpha)
sd(clt.alpha)^2

hist(clt.beta)
sd(clt.beta)^2
```