

# Economics 670: Econometrics

Department of Economics, Finance and Legal Studies

University of Alabama

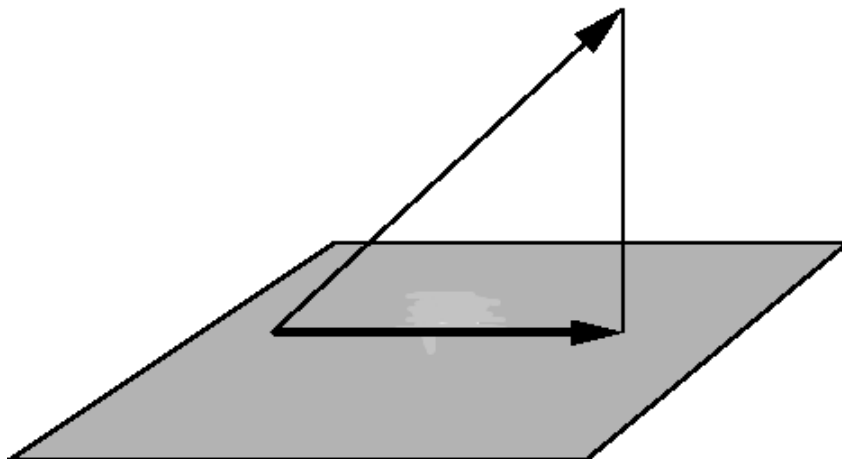
Fall 2020

Midterm I

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider two continuous random variables  $y$  and  $x$  that are each defined on the real number line (with no underlying assumption on their relationship). Noting that  $E(y|x) = \int yf(y|x)dy$  and  $E(y) = \int yf(y)dy$ , where  $f(y|x)$  is the conditional density of  $y$  given  $x$  and  $f(y)$  is the marginal density of  $y$ , prove the law of iterative expectations:  $E(E(y|x)) = E(y)$ .

2. Consider a linear projection of  $y$  on  $x$ . On the figure below, label the following: the column space of  $x$  (i.e.,  $col(x)$ ), the value of  $y$ , the fitted value of  $y$  (i.e.,  $\hat{y}$ ), and the residual (i.e.,  $\hat{\epsilon}$ ). Now, using the projection matrix  $P = x(x'x)^{-1}x'$  and the orthogonal projection matrix  $M = I - P$ , show that  $Py = \hat{y}$  and  $My = \hat{\epsilon}$ .



3. Consider the ordinary least-squares regression of  $y$  on  $x$  assuming homoskedasticity. Using the values in the output file below (and values that can be derived from the information below), give the formula for: the residuals ( $\hat{e}$ ), the residual sum of squares,  $R^2$ , OLS coefficient estimates ( $\hat{\beta}_0$  and  $\hat{\beta}_1$ ) and their respective standard errors.

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-0.0709	0.0188	-3.77	0.0002
x	0.1379	0.0262	5.26	0.0000

Mean of the dependent variable: 0.0000

Mean of the independent variable: 0.5144

Residual standard error: 0.9977 on 5793 degrees of freedom

Multiple R-squared: 0.0048, Adjusted R-squared: 0.0046