Economics 670: Econometrics

Department of Economics, Finance and Legal Studies

University of Alabama

Fall 2020

Midterm I

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider two continuous random variables y and x that are each defined on the real number line (with no underlying assumption on their relationship). Noting that $E(y|x) = \int yf(y|x)dy$ and $E(y) = \int yf(y)dy$, where $f(y|x)$ is the conditional density of y given x and $f(y)$ is the marginal density of y, prove the law of iterative expectations: $E\left(E(y|x)\right) = E(y)$.

$$E(E(y|x)) = \int E(y|x) f(x) dx$$

$$= \int \left[\int y f(y|x) dy \right] f(x) dx$$

$$= \int\int y f(y|x) f(x) dy dx$$

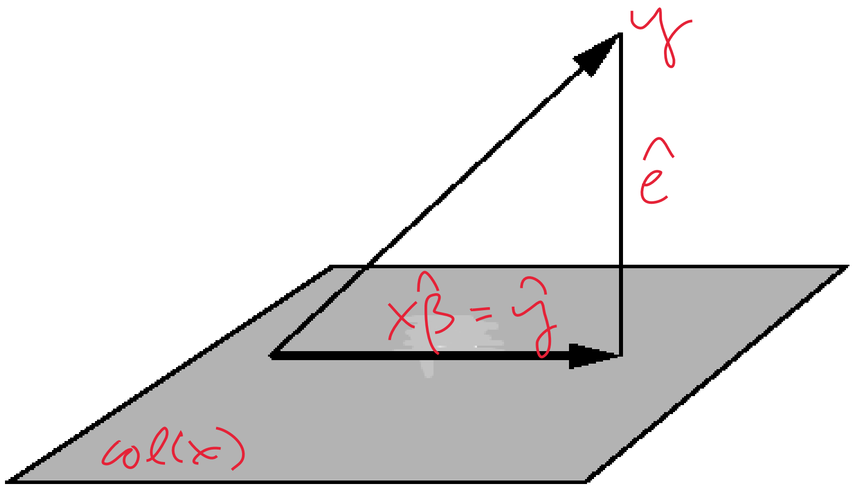
$$= \int\int y f(y,x) dy dx$$

$$= \int y \int f(y,x) dy dx$$

$$= \int y f(y) dy$$

$$= E(y)$$

2. Consider a linear projection of y on x. On the figure below, label the following: the column space of x (i.e., $col(x)$), the value of y, the fitted value of y (i.e., $\widehat{y}$), and the residual (i.e., $\widehat{e}$). Now, using the projection matrix $P = x(x'x)^{-1}x'$ and the orthogonal projection matrix $M = I - P$, show that $Py = \widehat{y}$ and $My = \widehat{e}$.



$$Py = x(x'x)^{-1}x'y = x\hat{\beta} = \hat{y}$$

$$\begin{aligned} My &= (I-P)y \\ &= y - x(x'x)^{-1}x'y \\ &= y - x\hat{\beta} \\ &= y - \hat{y} \\ &= \hat{e} \end{aligned}$$

3. Consider the ordinary least-squares regression of y on x assuming homoskedasticity. Using the values in the output file below (and values that can be derived from the information below), give the formula for: the residuals ($\hat{e}$), the residual sum of squares, R^2, OLS coefficient estimates ($\widehat{\beta}_0$ and $\widehat{\beta}_1$) and their respective standard errors.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.0709	0.0188	-3.77	0.0002
x	0.1379	0.0262	5.26	0.0000

Mean of the dependent variable: 0.0000

Mean of the independent variable: 0.5144

Residual standard error: 0.9977 on 5793 degrees of freedom

Multiple R-squared: 0.0048, Adjusted R-squared: 0.0046

$df = n - k, k = 2$

$n = 5795$

(a) $\hat{e}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i = y_i + 0.0709 - 0.1379 x_i$

(b) $\sum_{i=1}^{n} \hat{e}_i^2 = \sum_{i=1}^{n} (y_i + 0.0709 - 0.1379 x_i)^2$

(c) $R^2 = 1 - \sum_{i=1}^{n} \hat{e}_i^2 \Big/ \sum_{i=1}^{n} (y_i - \bar{y})^2 = 1 - (c) \Big/ \sum_{i=1}^{n} (y_i - 0.0000)^2$

(d) $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 0.0000 - 0.1379(0.5144)$

$\hat{\beta}_1 = \sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x}) \Big/ \sum_{i=1}^{n} (x_i - \bar{x})^2 = \dfrac{\sum_{i=1}^{n} (y_i - 0.000)(x_i - 0.5144)}{\sum_{i=1}^{n} (x_i - 0.5144)^2}$

(e) $V(\hat{\beta}) = s^2 (X'X)^{-1} \Rightarrow se(\hat{\beta}) = s\sqrt{(X'X)^{-1}} = 0.9977\sqrt{(X'X)^{-1}}$

where $X = (1 \; x)$

* alternative ways exist to obtain many of these values