Economics 670: Econometrics

Department of Economics, Finance and Legal Studies

University of Alabama

Fall 2019

Midterm I

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider the random variable X which follows a uniform distribution with support $x \in [a, b]$. Noting that the probability density function of a uniform random variable is $f_X(x) = \frac{1}{b-a}$ for $x \in [a, b]$ and zero otherwise, derive the first two centered moments (i.e., mean and variance). Without deriving the result, what is the value for third centered moment of X (i.e., skewness)?

2. Suppose we have data on 22 individuals, where $y = ln(income)$ and $x = ln(education)$:

$$\overline{y} = 20, \overline{x} = 10, \sum_{i=1}^{n}(y_i - \overline{y})^2, \sum_{i=1}^{n}(x_i - \overline{x})^2 = 60, \sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y}) = 30$$

(a) Compute the linear projection estimates of α and β in the model $y = \alpha + x\beta + e$

(b) Compute SST, SSR, SSE, R^2 and $\widehat{\sigma}^2$

3. Let $\widehat{\beta}_1$ and $\widehat{\beta}_2$ denote the estimated regression coefficients for the linear model $y = X_1\beta_1 + X_2\beta_2 + e$ where X_1 is a $n \times k_1$ matrix, X_2 is a $n \times k_2$ matrix and β_1 and β_2 are of dimensions k_1 and k_2, respectively. Show that $\widetilde{\beta}_1 = \widehat{\beta}_1 + (X_1'X_1)^{-1}X_1'X_2\widehat{\beta}_2$, where $\widetilde{\beta}_1$ is a vector of coefficients from the regression of y solely on X_1. Argue/show that $\widehat{\beta}_1$ is an unbiased estimator of β_1. Under what conditions is $\widetilde{\beta}_1$ an unbiased estimator for β_1. Note that $\widehat{\beta}_1 = \widetilde{\beta}_1 - (X_1'X_1)^{-1}X_1'X_2\widehat{\beta}_2 = (X_1'X_1)^{-1}X_1'(y - X_2\widehat{\beta}_2)$.