Economics 513: Economic Forecast and Analysis

Department of Economics, Finance and Legal Studies

University of Alabama

Spring 2022

Midterm II -Key

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider a pair of stationary time series variables $\{y_t, x_t\}_{t=1}^{T}$ that correspond to the set of equations below (where u_t is a generic, mean zero error). With this information, answer the following:

$$
\begin{aligned}
y_t &= 0.65 + 0.9y_{t-1} + u_t \\
y_t &= 0.55 + 0.8y_{t-2} + u_t \\
y_t &= 0.45 + 0.7y_{t-3} + u_t \\
y_t &= 0.35 + 0.6y_{t-4} + u_t \\
y_t &= 0.25 + 0.5y_{t-5} + u_t
\end{aligned}
$$

$$
\begin{aligned}
y_t &= 0.65 + 0.9y_{t-1} + u_t \\
y_t &= 0.55 + 0.5y_{t-1} + 0.0y_{t-2} + u_t \\
y_t &= 0.45 + 0.5y_{t-1} + 0.4y_{t-2} + 0.0y_{t-3} + u_t \\
y_t &= 0.35 + 0.5y_{t-1} + 0.4y_{t-2} + 0.3y_{t-3} + 0.0y_{t-4} + u_t \\
y_t &= 0.25 + 0.5y_{t-1} + 0.4y_{t-2} + 0.3y_{t-3} + 0.2y_{t-4} + 0.0y_{t-5} + u_t
\end{aligned}
$$

$$
\begin{aligned}
y_t &= 0.25 + 0.0x_t + u_t \\
y_t &= 0.35 + 0.0x_{t-1} + u_t \\
y_t &= 0.45 + 0.5x_{t-2} + u_t \\
y_t &= 0.55 + 0.0x_{t-3} + u_t \\
y_t &= 0.65 + 0.0x_{t-4} + u_t
\end{aligned}
$$

   (a) Plot the sample autocorrelation function (ACF).

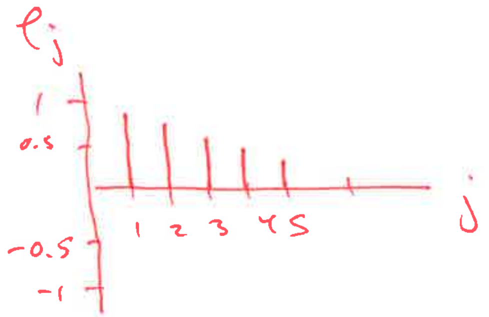
   (b) Plot the sample partial autocorrelation function (PACF).

   (c) Plot the sample cross-correlation function (CCF).

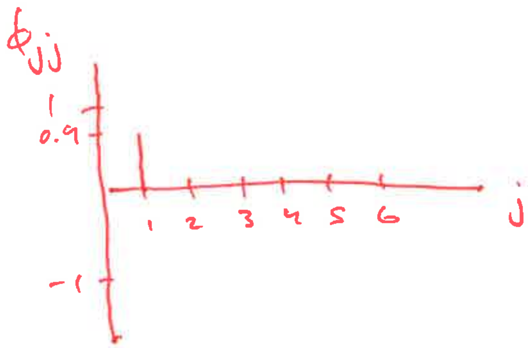
   (d) Given (a-c), write down your tentative model.

   (e) For your model in part (d), write down the log-likelihood function.

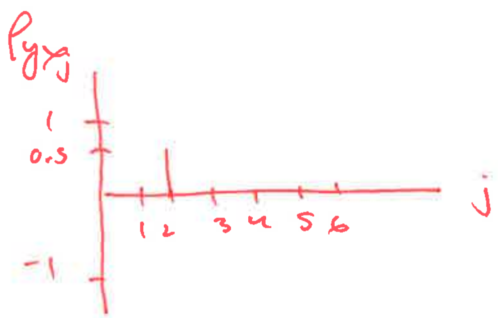
(a)



(b)



(c)



(d) $y_t = c + \phi y_{t-1} + \beta x_{t-2} + \varepsilon_t, \quad \varepsilon_t \sim WN($

(e) $$\ln \mathcal{L}(c, \phi, \beta, \sigma^2) = -\frac{(T-3)}{2}\ln 2\pi - \frac{(T-3)}{2}\ln \sigma^2 - \frac{1}{2\sigma^2}\sum_{t=3}^{T}\left(y_t - c - \phi y_{t-1} - \beta x_{t-2}\right)^2$$

2. Consider the following model: $Y_t = \mu + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \theta_3\varepsilon_{t-3}$, where ε_t is a white noise sequence, $N(0, \sigma^2)$. With this information, answer the following:

(a) Find the h-step ahead forecast for $h = 1, 2, \ldots$

(b) Find the h-step ahead forecast error for $h = 1, 2, \ldots$

(c) Find the h-step ahead forecast error variance for $h = 1, 2, \ldots$

(d) Find the h-step ahead interval forecast for $h = 1, 2, \ldots$

(e) Find the h-step ahead density forecast for $h = 1, 2, \ldots$

$$Y_{t+h} = \mu + \varepsilon_{t+h} + \theta_1 \varepsilon_{t+h-1} + \theta_2 \varepsilon_{t+h-2} + \theta_3 \varepsilon_{t+h-3}$$

(a) $$\hat{Y}_{t+1|t} = E(Y_{t+1}|\Omega_t) = E(\mu + \varepsilon_{t+1} + \theta_1\varepsilon_t + \theta_2\varepsilon_t + \theta_3\varepsilon_{t-2}|\Omega_t)$$

$$= \mu + \theta_1\varepsilon_t + \theta_2\varepsilon_{t-1} + \theta_3\varepsilon_{t-2}$$

$$\hat{Y}_{t+2|t} = \mu + \theta_2\varepsilon_t + \theta_3\varepsilon_{t-1}$$

$$\hat{Y}_{t+3|t} = \mu + \theta_3\varepsilon_t$$

$$\hat{Y}_{t+h|t} = \mu \quad \forall h > 3$$

(b) $$e_{t+h} = Y_{t+h} - \hat{Y}_{t+h|t}$$

$$e_{t+1} = Y_{t+1} - Y_{t+1|t} = \varepsilon_{t+1}$$

$$e_{t+2} = \varepsilon_{t+2} + \theta_1\varepsilon_{t+1}$$

$$e_{t+3} = \varepsilon_{t+3} + \theta_1\varepsilon_{t+2} + \theta_2\varepsilon_{t+1}$$

$$e_{t+h} = \varepsilon_{t+h} + \theta_1\varepsilon_{t+h-1} + \theta_2\varepsilon_{t+h-2} + \theta_3\varepsilon_{t+h-3}$$

$$\forall h > 3$$

(c) $V(e_{t+h})$

$V(e_{t+1}) = V(\varepsilon_{t+1}) = \sigma^2$

$V(e_{t+2}) = (1+\theta_1^2)\sigma^2$

$V(e_{t+3}) = (1+\theta_1^2+\theta_2^2)\sigma^2$

$V(e_{t+h}) = (1+\theta_1^2+\theta_2^2+\theta_3^2)\sigma^2 \quad \forall h > 3$

(d) $\left[\hat{Y}_{t+h|t} \pm 1.96\sqrt{V(e_{t+h})}\right]$

plug in from parts (a) & (b)

(e) $N(\hat{Y}_{t+h|t}, V(e_{t+h}))$

plug in from parts (a) & (b)

3. Consider the two pieces of R output listed below. We are interested in testing the null hypothesis that the second lag of ε is zero in the second model: $H_0 : \theta_2 = 0$ vs $H_1 : \theta_2 \neq 0$. With this information, answer the following:

model.fit = arima(data,order=c(1,0,1),method='ML')
model.fit
Coefficients:

	ar1	ma1	intercept
	0.5165	-0.0133	0.0074
s.e.	0.2400	0.2749	1.9454

SSR 407.5: log likelihood = -140.87, aic = 289.75, bic = 296.79

model.fit = arima(data,order=c(1,0,2),method='ML')
model.fit
Coefficients:

	ar1	ma1	ma2	intercept
	0.0488	0.4705	0.2496	-0.0190
s.e.	0.7467	0.7376	0.3428	1.7343

SSR 405.1: log likelihood = -140.76, aic = 291.53, bic = 300.33

(a) Write down the log-likelihood function for the null model.

(b) Write down the log-likelihood function for the alternative model.

(c) Use a t-test to test the null hypothesis.

(d) Use a likelihood-ratio test to test the null hypothesis.

(e) Use a Lagrange-multiplier test to test the null hypothesis.

(a) $$\ln \mathcal{L}(\phi, c, \theta_1, \sigma^2) = -\frac{(T-2)}{2}\ln 2\pi - \frac{(T-2)}{2}\ln \sigma^2 - \frac{1}{2\sigma^2}\sum_{t=3}^{T}(y_t - c - \phi y_{t-1} - \theta_1 \varepsilon_{t-1})^2$$

(b) $$\ln \mathcal{L}(\phi, c, \theta_1, \theta_2, \sigma^2) = -\frac{(T-2)}{2}\ln 2\pi - \frac{(T-2)}{2}\ln \sigma^2 - \frac{1}{2\sigma^2}\sum_{t=3}^{T}(y_t - c - \phi y_{t-1} - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2})^2$$

(c) $H_0 : \theta_2 = 0$ vs. $H_1 : \theta_2 \neq 0$

$$t = \frac{0.2496 - 0}{0.3428} < 2 \Rightarrow \text{fail to reject } H_0$$

(d) $LR = -2\ln\lambda = -2\ln\left(\frac{\mathcal{L}(\theta_0)}{\mathcal{L}(\theta_1)}\right)$

$$= -2\ln\left(\frac{-140.87}{-140.76}\right) \approx 0$$

$\Rightarrow$ fail to reject H_0

(e) $LM = l \cdot F$

$$= 1 \cdot \frac{(SSR_R - SSR_u)/1}{SSR_u/(n-4)}$$

$$= \frac{(407.5 - 405.1)/1}{(405.1)/(n-4)} \approx 0$$

$\Rightarrow$ fail to reject H_0