Economics 513: Economic Forecast and Analysis

Department of Economics, Finance and Legal Studies
University of Alabama
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Midterm II — Key

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider the following data generating process

$$Y_t = (-1)^t \varepsilon_t$$

where ε_t is a white noise sequence. For the process Y_t, answer the following:

(a) Find the h-step ahead forecast for $h = 1, 2, \ldots$.

(b) Find the h-step ahead forecast error for $h = 1, 2, \ldots$

(c) Find the h-step ahead interval forecast for $h = 1, 2, \ldots$

(d) Find the h-step ahead density forecast for $h = 1, 2, \ldots$

(e) For *a* specific realization of Y_t and *a* specific value of h, plot y_t along with (a), (c) and (d) in a single figure.

(a) $\hat{Y}_{t+h|t} = E(Y_{t+h}|\Omega_t) = E[(-1)^{t+h}\varepsilon_{t+h}|\Omega_t]$

$= (-1)^{t+h}E(\varepsilon_{t+h}|\Omega_t) = 0 \ \forall h > 0$

(b) $e_{t+h} = Y_{t+h} - \hat{Y}_{t+h|t} = (-1)^{t+h}\varepsilon_{t+h} - 0$

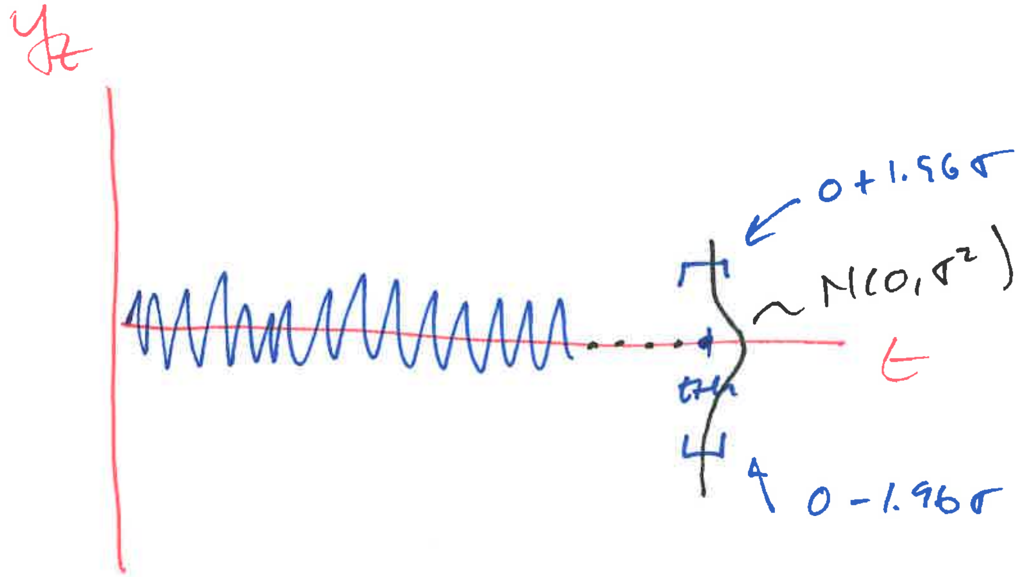
$= (-1)^{t+h}\varepsilon_{t+h} \ \forall h$

(c) $V(e_{t+h}) = V[(-1)^{t+h}\varepsilon_{t+h}] = (-1)^{2(t+h)}V(\varepsilon_{t+h}) = \sigma^2$

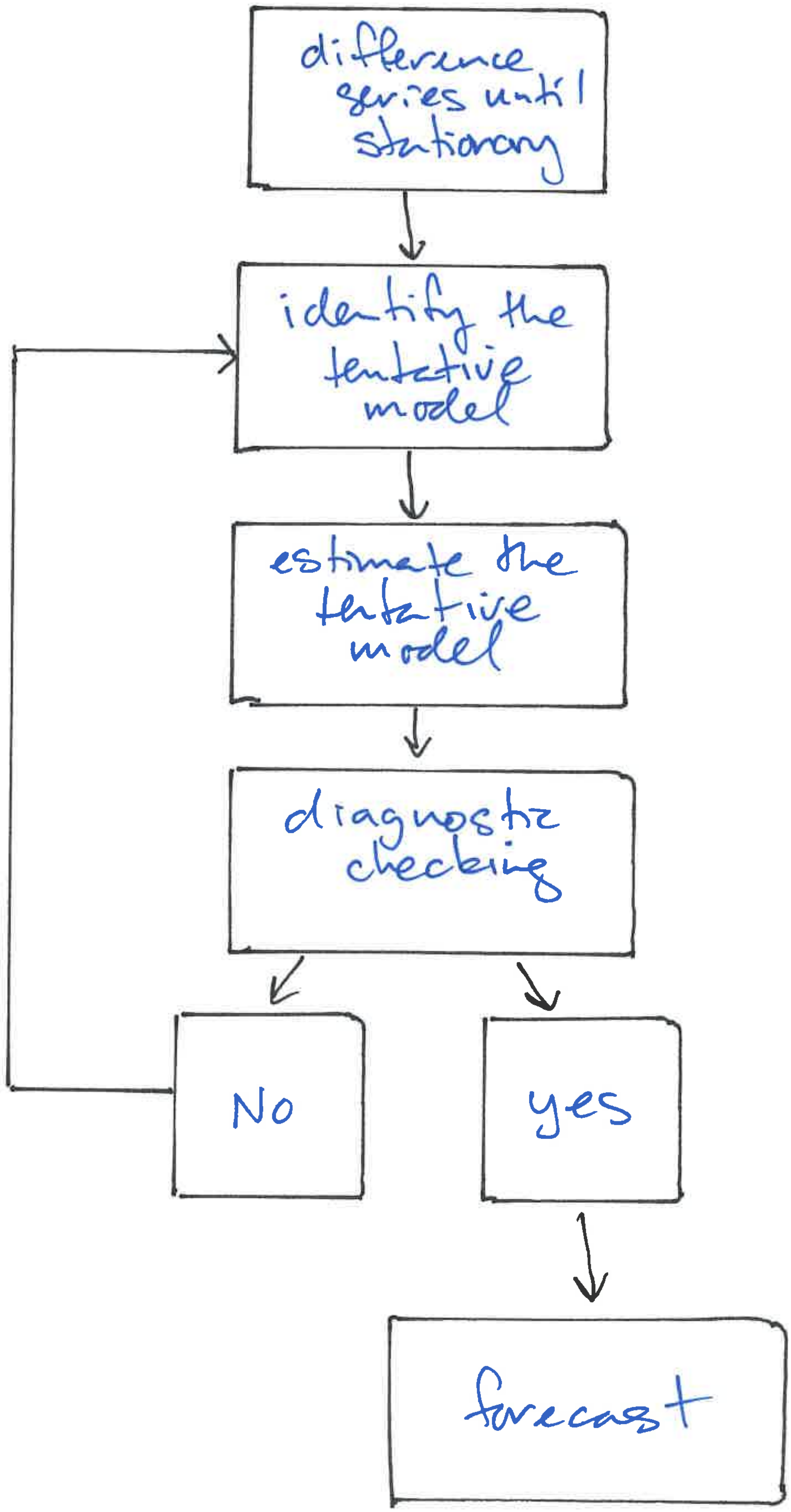
$[0 \pm 1.96\sigma] \ \forall h$

(d) $N(0, \sigma^2)$

(e)



2. Draw the flow chart for the Box-Jenkins methodology. Below the chart write one or two sentences to describe each step.



- check if series is stationary
  make so if not
- use ACF & PACF to make
  educated guess at model
- use MLE to estimate parameters
  of the model
- check if residuals are white
  noise (ACF, PACF, Boxtest, etc.)
- if no, start w/ new model
- if yes, move on
- predict y_t in period $t+h$

3. Consider the pieces of R output listed below and suppose the last two values for y_t are $y_T = 0.50$ and $y_{T-1} = 1.50$, respectively, where $t = 1, 2, \ldots, T$. With this information, answer the following:

model.fit = arima(data,order=c(1,0,2),method='ML')

model.fit

Coefficients:

	ar1 $\hat\phi$	ma1 $\hat\theta_1$	ma2 $\hat\theta_2$	intercept $\hat c$
	0.0488	0.4705	0.2496	-0.0190
s.e.	0.7467	0.7376	0.3428	1.7343

SSR 405.1: log likelihood = -140.76, aic = 291.53, bic = 300.33

resid = model.fit$residuals

data = resid

resid.fit = arima(data,order=c(3,0,0),method='ML')

resid.fit

Coefficients:

	ar1 $\hat\rho_1$	ar2 $\hat\rho_2$	ar3 $\hat\rho_3$	intercept $\hat\delta$
	0.0345	0.0590	0.0459	-0.0190
s.e.	0.7467	0.7376	0.3428	1.7343

SSR 405.1: log likelihood = -140.76, aic = 291.53, bic = 300.33

(a) Write the equation for the first model?

(b) Derive the one-step-ahead forecast from the first model.

(c) What is the h-step ahead forecast for the first model when h tends to infinity?

(d) Write the equation for the second model?

(e) What can you conclude from the second model?

(a) $y_t = c + \phi y_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$

$\hat{y}_t = \hat{c} + \hat{\phi} y_{t-1} + \hat{\theta}_1 \hat{\varepsilon}_{t-1} + \hat{\theta}_2 \hat{\varepsilon}_{t-2}$

(b) $\hat{y}_{T+1|T} = \hat{c} + \hat{\phi} y_T + \hat{\theta}_1 \hat{\varepsilon}_{T} + \hat{\theta}_2 \hat{\varepsilon}_{T-1}$

(c) $\hat{y}_{T+h|T} \to \dfrac{\hat{c}}{1-\hat{\phi}}$ as $h \to \infty$ since $|\hat{\phi}| < 1$

(d) $\hat{\varepsilon}_t = \hat{\delta} + \hat{\rho}_1 \hat{\varepsilon}_{t-1} + \hat{\rho}_2 \hat{\varepsilon}_{t-2} + \hat{\rho}_3 \hat{\varepsilon}_{t-3}$

(e) each $\hat{\rho}_i$ is insignificant this suggests no autocorrelation, but not a formal test