Economics 513: Economic Forecast and Analysis

Department of Economics, Finance and Legal Studies

University of Alabama

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Midterm II — Key

The exam consists of four questions on four pages. Each question is of equal value.

1. Consider the model $y_t = c + \phi y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$, where $\varepsilon_t \sim WN\left(0, \sigma_\varepsilon^2\right)$ is a white noise sequence.

   (a) Write the log-likelihood function needed to estimate this model.

   (b) Consider the null hypothesis $H_0 : \phi = 0$. Write the log-likelihood function needed to estimate the model under the null hypothesis.

   (c) Write the likelihood ratio test relevant for the test in part (b).

   (d) Consider the null hypothesis $H_0 : \theta = \phi = 0$. Write the log-likelihood function needed to estimate the model under the null hypothesis.

   (e) For the objective function in part (d), derive the estimator of c.

(a) $\ln \mathcal{L}(c, \phi, \theta, \sigma^2) = -\frac{T}{2}\ln 2\pi - \frac{T}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum_{t=2}^{T}\varepsilon_t^2$

$\varepsilon_t = y_t - c - \phi y_{t-1} - \theta\varepsilon_{t-1}$

(b) $\ln \mathcal{L}(c, \theta, \sigma^2) = -\frac{T}{2}\ln 2\pi - \frac{T}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum_{t=1}^{T}\varepsilon_t^2$

$\varepsilon_t = y_t - c - \theta\varepsilon_{t-1}$

(c) $H_0: \phi = 0$ vs. $H_1: \phi \neq 0$

$LR = -2\left(\ln \mathcal{L}(c, \theta, \sigma^2) - \ln(c, \phi, \theta, \sigma^2)\right) \sim \chi_1^2$

(d) $\ln \mathcal{L}(c, \sigma^2) = -\frac{T}{2}\ln 2\pi - \frac{T}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum_{t=1}^{T}\varepsilon_t^2$

$\varepsilon_t = y_t - c$

(e) $\frac{\partial \ln \mathcal{L}(c, \sigma^2)}{\partial c} = \frac{-1}{\sigma^2}\sum_{t=1}^{T}(y_t - c) = 0 \Rightarrow$

$\hat{c} = \frac{1}{T}\sum_{t=1}^{T} y_t = \bar{y}$

2. Consider two processes Y_t and X_t. Suppose that Y_t represents yearly personal consumption expenditures on food and X_t represents disposable personal income. Suppose we are interested in eventually forecasting Y with the help of both past values of Y as well as past values of X. With this information, answer the following questions

   (a) Write down an ARMA(0,1) which also includes a (single) first lagged value for X

   (b) How do you know how many lags to include for Y? How do you know how many lags to include for ε (the error term)? How do you know how many lags to include for X?

   (c) Write down the h-step ahead value for Y (Y_{t+h}) that you put down in part (a)

   (d) Construct the forecast value of Y ($\widehat{Y}_{t+h|t}$) for $h = 1$

   (e) Why is it difficult to forecast for $h = 2$ and beyond?

(a) $Y_t = c + \beta_1 X_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$

(b) ACF (for ε), PACF (for Y), CCF (for X)

(c) $Y_{t+h} = c + \beta_1 X_{t+h-1} + \varepsilon_{t+h} + \theta\varepsilon_{t+h-1}$

(d) $Y_{t+1} = c + \beta_1 X_t + \varepsilon_{t+1} + \theta\varepsilon_t$

$\hat{Y}_{t+1|t} = E(Y_{t+1} | \Omega_t)$

$= E(c + \beta_1 X_t + \varepsilon_{t+1} + \theta\varepsilon_t | \Omega_t)$

$= c + \beta_1 X_t + \theta\varepsilon_t$

(e) $\hat{Y}_{t+2|t}$ & beyond require forecasts of X_t, for example

$\hat{Y}_{t+2|t} = E(Y_{t+2} | \Omega_t)$

$= c + \beta_1 E(X_{t+1} | \Omega_t)$

3. Suppose the true data generating process is $y_t = c + \phi y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$, but you choose your tentative model to be $y_t = c + \phi y_{t-1} + \varepsilon_t$. Assuming that $|\phi| < 1$, answer the following:

   (a) Write both the likelihood function for the true model and for your model.

   (b) Draw a sample ACF and PACF for the *residuals* from both the true model and for your model.

   (c) What will the null hypothesis and conclusion from the Box and Pierce test (Q-statistic) be for both the true model and for your model?

   (d) What is the consequence from choosing the wrong model for point forecasts (for $h = 1,$ [redacted])?

   (e) What is the consequence from choosing the wrong model for interval forecasts (for $h = 1$ [redacted])?

(a) $\ln \mathcal{L}(c, \phi, \varepsilon_t) = -\frac{T}{2}\ln 2\pi - \frac{T}{2}\ln \sigma^2 - \frac{1}{2\sigma^2}\sum_{t=2}^{T}\varepsilon_t^2$

$\varepsilon_t = y_t - c - \phi y_{t-1} - \theta\varepsilon_{t-1}$

$\ln \mathcal{L}(c, \phi, \sigma^2) = -\frac{T}{2}\ln 2\pi - \frac{T}{2}\ln \sigma^2 - \frac{1}{2\sigma^2}\sum_{t=2}^{T}\varepsilon_t^2$

$\varepsilon_t = y_t - c - \phi y_{t-1}$

(b) ACF & PACF should look like WN for the true model & an AR(1) for the residuals of the tentative model

(c) The null in each case is no serial correlation.
We should fail to reject in the true model & reject in the tentative model

(d) point forecast will be off by $\hat{\theta}\hat{\varepsilon}_t$ for $h=1$

(e) ~~the variance will be too small for the tentative model as it omits the variation from~~ $\cancel{\theta\varepsilon_{t-1}}$

it will be the same for $h=1$
$V(e_{t+1}) = \sigma^2$

4. Consider the R output listed below. With this information, answer the following:

model.fit = arima(data,order=c(0,0,2),method='ML')

model.fit

Coefficients:

	ma1	ma2	intercept
	0.4705	0.2496	-0.0190
s.e.	0.7376	0.3428	1.7343

sigma2 estimated as 40.51: log likelihood = -140.76, AIC = 291.53, AICc = 293.15, BIC = 300.33

(a) Find the h-step ahead forecast for $h = 1, 2, \ldots$

(b) Find the h-step ahead forecast error for $h = 1, 2, \ldots$

(c) Find the h-step ahead forecast error variance for $h = 1, 2, \ldots$

(d) Find the h-step ahead forecast interval forecast for $h = 1, 2, \ldots$

(e) Plots parts (a) and (d) in a single figure

$$y_t = -0.0190 + \varepsilon_t + 0.4705\,\varepsilon_{t-1} + 0.2496\,\varepsilon_{t-2}$$

(a) $$\hat{y}_{t+1|t} = -0.0190 + \hat{\varepsilon}_{t+1|t} + 0.4705\,\hat{\varepsilon}_t + 0.2496\,\hat{\varepsilon}_{t-1}$$

$$= -0.0190 + 0.4705\,\hat{\varepsilon}_t + 0.2496\,\hat{\varepsilon}_{t-1}$$

$$\hat{y}_{t+2|t} = -0.0190 + 0.2496\,\hat{\varepsilon}_t$$

$$\hat{y}_{t+j|t} = -0.0190 \quad \forall h > 2$$

(b) $$e_{t+h} = y_{t+h} - \hat{y}_{t+h|t}$$

$$e_{t+1} = y_{t+1} - \hat{y}_{t+1|t}$$

$$= \varepsilon_{t+1}$$

$$e_{t+2} = y_{t+2} - \hat{y}_{t+2|t}$$

$$= \varepsilon_{t+2} + 0.4705\,\varepsilon_{t+1}$$

$$e_{t+3} = \varepsilon_{t+3} + 0.4705\,\varepsilon_{t+2} + 0.2496\,\varepsilon_{t+1}$$

$$e_{t+h} = \varepsilon_{t+h} + 0.4705\,\varepsilon_{t+h-1} + 0.2496\,\varepsilon_{t+h-2} \quad \forall h > 2$$

(c) $V(e_{t+h}) = \; ?$

$$V(e_{t+1}) = V(\varepsilon_{t+1}) = \hat{\sigma}^2$$

$$V(e_{t+2}) = V(\varepsilon_{t+2} + 0.4705\,\varepsilon_{t+1}) = \hat{\sigma}^2(1 + 0.4705^2)$$

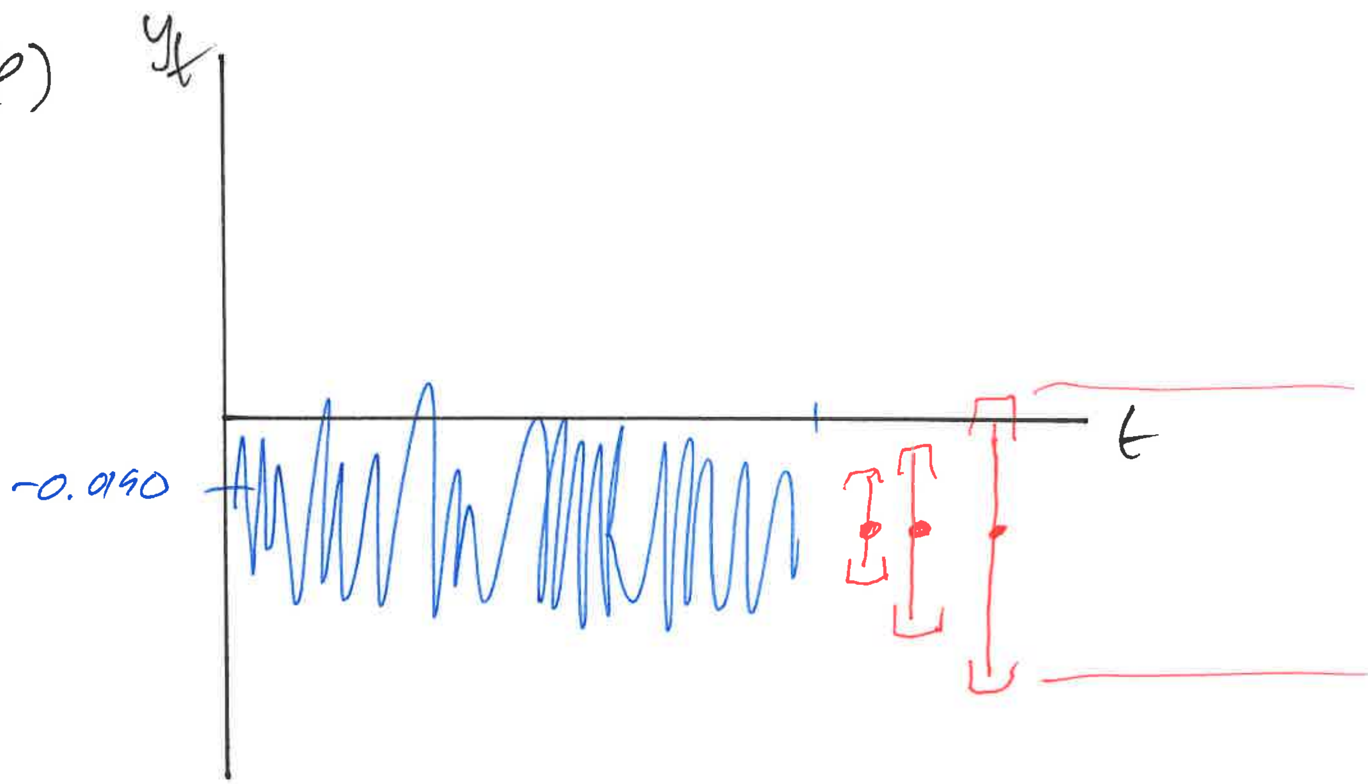
$$V(e_{t+3}) = V(\varepsilon_{t+3} + 0.4705\,\varepsilon_{t+2} + 0.2496\,\varepsilon_{t+1})$$
$$= \hat{\sigma}^2(1 + 0.4705^2 + 0.2496^2)$$

$$V(e_{t+h}) = \hat{\sigma}^2(1 + 0.4705^2 + 0.2496^2) \quad \forall h > 2$$

(d) $\left[\hat{y}_{t+h|t} \pm 1.96\sqrt{V(e_{t+h})}\right]$

plug in the relevant values

(e)



point forecasts in part (a)
interval forecasts in part (d)