Economics 513: Economic Forecast and Analysis

Department of Economics, Finance and Legal Studies

University of Alabama

Spring 2022

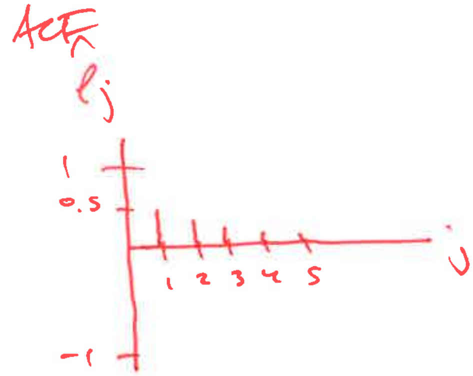
Midterm I Key

The exam consists of three questions on four pages. Each question is of equal value.

1. Consider each set of equations below. For each set, plot the *sample* ACF *or* PACF that corresponds to the set of equations.

   (a)

$$y_t = 0.25 + 0.5y_{t-1} + u_t$$
$$y_t = 0.35 + 0.4y_{t-2} + u_t$$
$$y_t = 0.45 + 0.3y_{t-3} + u_t$$
$$y_t = 0.55 + 0.2y_{t-4} + u_t$$
$$y_t = 0.65 + 0.1y_{t-5} + u_t$$



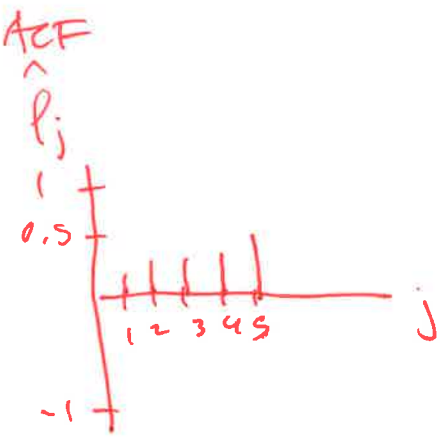
   (b)

$$y_t = 0.65 + 0.5y_{t-1} + u_t$$
$$y_t = 0.55 + 0.4y_{t-2} + u_t$$
$$y_t = 0.45 + 0.3y_{t-3} + u_t$$
$$y_t = 0.35 + 0.2y_{t-4} + u_t$$
$$y_t = 0.25 + 0.1y_{t-5} + u_t$$

identical to (a)

   (c)

$$y_t = 0.65 + 0.1y_{t-1} + u_t$$
$$y_t = 0.55 + 0.2y_{t-2} + u_t$$
$$y_t = 0.45 + 0.3y_{t-3} + u_t$$
$$y_t = 0.35 + 0.4y_{t-4} + u_t$$
$$y_t = 0.25 + 0.5y_{t-5} + u_t$$



(d)

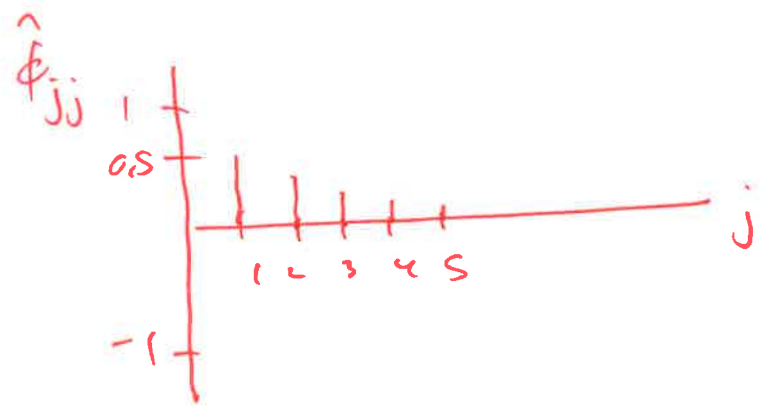
$$y_t = 0.65 + 0.5y_{t-1} + u_t$$
$$y_t = 0.55 + 0.5y_{t-1} + 0.4y_{t-2} + u_t$$
$$y_t = 0.45 + 0.5y_{t-1} + 0.4y_{t-2} + 0.3y_{t-3} + u_t$$
$$y_t = 0.35 + 0.5y_{t-1} + 0.4y_{t-2} + 0.3y_{t-3} + 0.2y_{t-4} + u_t$$
$$y_t = 0.25 + 0.5y_{t-1} + 0.4y_{t-2} + 0.3y_{t-3} + 0.2y_{t-4} + 0.1y_{t-5} + u_t$$

(e)

$$y_t = 0.25 + 0.5y_{t-1} + u_t$$
$$y_t = 0.35 + 0.5y_{t-1} + 0.0y_{t-2} + u_t$$
$$y_t = 0.45 + 0.5y_{t-1} + 0.4y_{t-2} + 0.3y_{t-3} + u_t$$
$$y_t = 0.55 + 0.5y_{t-1} + 0.4y_{t-2} + 0.3y_{t-3} + 0.0y_{t-4} + u_t$$
$$y_t = 0.65 + 0.5y_{t-1} + 0.4y_{t-2} + 0.3y_{t-3} + 0.2y_{t-4} + 0.1y_{t-5} + u_t$$

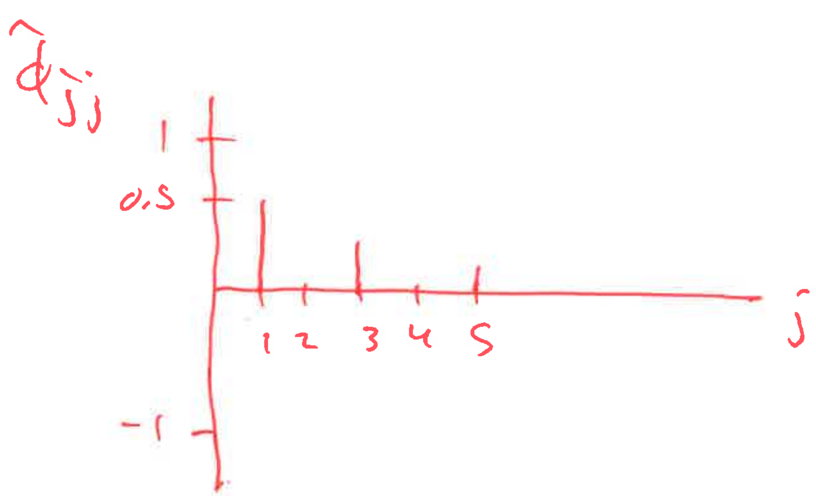
PACF

(d)



PACF

(e)



2. Consider the following model: $Y_t = 1 + \varepsilon_t + \frac{1}{2}\varepsilon_{t-1} + \frac{1}{4}\varepsilon_{t-2}$, where $\varepsilon_t \sim WN$

(a) Derive the expected value of the series.

(b) Derive the variance of the series.

(c) Derive the autocovariance of the series for all lags $j = 1, 2, \ldots$.

(d) Derive the autocorrelation of the series for all lags $j = 1, 2, \ldots$.

(e) Plot the autocorrelation function.

(a) $E(Y_t) = E(1 + \varepsilon_t + \frac{1}{2}\varepsilon_{t-1} + \frac{1}{4}\varepsilon_{t-2}) = 1$

(b) $V(Y_t) = \gamma_0 = E[(Y_t - \mu)^2]$

$= E[(\varepsilon_t + \frac{1}{2}\varepsilon_{t-1} + \frac{1}{4}\varepsilon_{t-2})^2]$

$= \sigma^2 + \frac{1}{4}\sigma^2 + \frac{1}{16}\sigma^2$

$= \sigma^2(1 + \frac{1}{4} + \frac{1}{16}) = \frac{21}{16}\sigma^2$

(c) $\gamma_j = E\{(Y_t - \mu)(Y_{t-j} - \mu)\}$

$\gamma_1 = E[(\varepsilon_t + \frac{1}{2}\varepsilon_{t-1} + \frac{1}{4}\varepsilon_{t-2})(\varepsilon_{t-1} + \frac{1}{2}\varepsilon_{t-2} + \frac{1}{4}\varepsilon_{t-3})]$

$= \frac{1}{2}\sigma^2 + \frac{1}{8}\sigma^2 = \frac{5}{8}\sigma^2$

$\gamma_2 = E[(\varepsilon_t + \frac{1}{2}\varepsilon_{t-1} + \frac{1}{4}\varepsilon_{t-2})(\varepsilon_{t-2} + \frac{1}{2}\varepsilon_{t-3} + \frac{1}{4}\varepsilon_{t-4})]$

$= \frac{1}{4}\sigma^2$

$\gamma_3 = 0, \ldots \quad \forall j > 2$

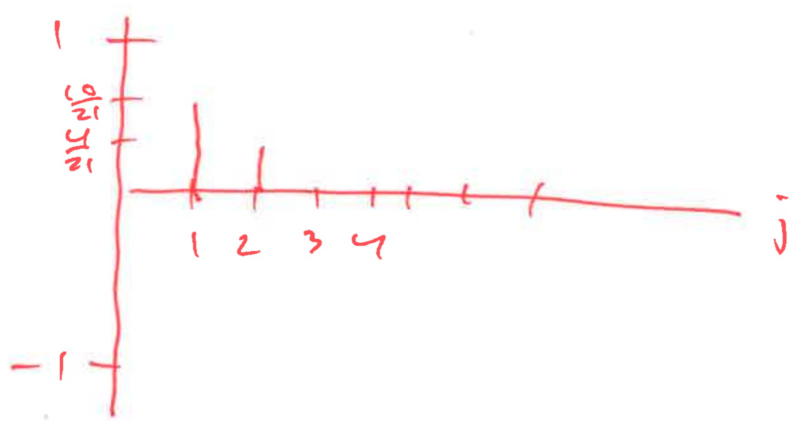
(d) $\rho_j = \frac{r_j}{r_0}$

$$\rho_1 = \frac{r_1}{r_0} = \frac{\frac{5}{8}\sigma^2}{\frac{21}{16}\sigma^2} = \frac{10}{21}$$

$$\rho_2 = \frac{r_2}{r_0} = \frac{\frac{1}{4}\sigma^2}{\frac{21}{16}\sigma^2} = \frac{4}{21}$$

$$\rho_3 = 0 \quad , \ldots \quad \forall j > 2$$

(e) ρ_j



3. Consider the two pieces of R output listed below. With this information, answer the following:

```
model.fit = arima(data,order=c(1,0,1),method='ML')
model.fit
Coefficients:
          ar1       ma1      intercept
        0.5165    -0.0133     0.0074
s.e.    0.2400     0.2749     1.9454
sigma2 estimated as 40.75: log likelihood = -140.87, aic = 289.75, bic = 296.79

model.fit = arima(data,order=c(1,0,2),method='ML')
model.fit
Coefficients:
          ar1       ma1       ma2      intercept
        0.0488     0.4705    0.2496     -0.0190
s.e.    0.7467     0.7376    0.3428      1.7343
sigma2 estimated as 40.51: log likelihood = -140.76, aic = 291.53, bic = 300.33
```

(a) What is the common name for each model? Write down each model (using the relevant estimates from the ouput listed above).

(b) Is there evidence to suggest that either model is stationary? Explain.

(c) Separately test the null hypothesis that each series is an MA process (write down the null and alternative hypothesis, test statistic, and decision rule).

(d) Test the null hypothesis that the first model is correctly specified versus the alternative hypothesis that the second model is correctly specified (write down the null and alternative hypothesis, test statistic, and decision rule).

(e) Use three of the model selection criteria listed in the output. For each selection criteria, which model does the criteria suggest?

(a) ARMA(1,1)

$y_t = 0.5165\, y_{t-1} - 0.0133\, \varepsilon_{t-1} + \varepsilon_t + 0.0074$

ARMA(1,2)

$y_t = 0.0488\, y_{t-1} + 0.4705\, \varepsilon_{t-1} + 0.2496\, \varepsilon_{t-2} + \varepsilon_t - 0.0190$

(b) $|\phi| < 1$ in each model & hence both are stationary

(c) $H_0: \phi = 0$

$H_a: \phi \neq 0$

$$t = \frac{\hat{\phi} - 0}{se(\hat{\phi})} = \begin{cases} \frac{0.5165 - 0}{0.2400} > 2 & \text{reject} \\ \frac{0.0488 - 0}{0.7467} < 2 & \text{fail to reject} \end{cases}$$

(d) $H_0: \theta_2 = 0$

$H_a: \theta_2 \neq 0$

$$t = \frac{\hat{\theta}_2 - 0}{se(\hat{\theta}_2)} = \frac{0.2496 - 0}{0.3428} < 2$$

fail to reject H_0

(e) $\hat{\sigma}^2 \Rightarrow$ model 2

$LL \Rightarrow$ model 2

$AIC \Rightarrow$ model 1

$BIC \Rightarrow$ model 1