Economics 513: Economic Forecast and Analysis

Department of Economics, Finance and Legal Studies

University of Alabama

Fall 2022

Midterm I — Key

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider the following data generating process

$$Y_t = (-1)^t \varepsilon_t$$

where ε_t is a white noise sequence. For the process Y_t, answer the following (show your work):

(a) Derive the expected value of Y_t (i.e., $E(Y_t) \equiv \mu$).

(b) Derive the variance of Y_t (i.e., $V(Y_t) \equiv \gamma_0$).

(c) Derive the covariance between Y_t and Y_{t-j} (i.e., $COV(Y_t, Y_{t-j}) \equiv \gamma_j$) for $j = 1, 2, \ldots$.

(d) Derive the autocorrelation function of this process (i.e., $CORR(Y_t, Y_{t-j}) \equiv \rho_j$) for $j = 1, 2, \ldots$.

(e) Plot the autocorrelation function you obtained in part (d).

(a) $E(Y_t) = E\left[(-1)^t \varepsilon_t\right] = (-1)^t E(\varepsilon_t) = 0$

(b) $V(Y_t) = E\left\{\left[(-1)^t \varepsilon_t - 0\right]^2\right\} = E\left[(-1)^{2t} \varepsilon_t^2\right]$

$= (-1)^{2t} E(\varepsilon_t^2) = 1 \cdot \sigma^2 = \sigma^2$

(c) $COV(Y_t, Y_{t-j}) = E\left\{\left[(-1)^t \varepsilon_t - 0\right]\left[(-1)^{t-j} \varepsilon_{t-j} - 0\right]\right\}$

$= E\left[(-1)^{-j} \varepsilon_t \varepsilon_{t-j}\right] = (-1)^j E(\varepsilon_t \varepsilon_{t-j}) = 0 \ \forall j > 0$

(d) $CORR(Y_t, Y_{t-j}) = \gamma_j / \gamma_0 = 0 \ \forall j > 0$

(e) $\rho_0 = 1, \ \rho_j = 0 \ \forall j > 0 \Rightarrow$ no spikes in ACF

2. In the figure below, we have four realizations of a stochastic process. With this information, answer the following:

   (a) Label the axes.

   (b) Write the formula to calculate the ensemble average for the year 1960 (i.e., the expectation of the stochastic process in the year 1960). Give a reasonable estimate of this average.

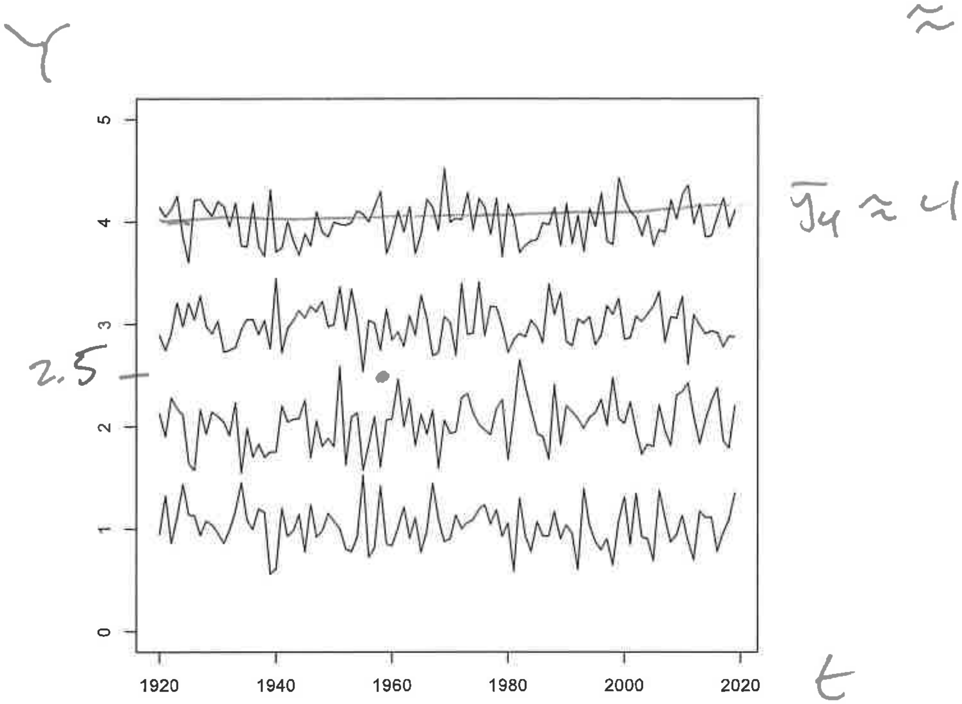
   (c) Write the formula to calculate the time series average for *a* realization of the stochastic process. For one of the realizations, give a reasonable estimate of this average.

   (d) Write the formula to calculate the variance of the stochastic process for the year 1960 (i.e., the variance of the stochastic process in the year 1960). Give a reasonable estimate of this variance?

   (e) Write the formula to calculate the variance of the time series for *a* realization of the stochastic variance. For one of the realizations, give a reasonable estimate of this variance.

(e) $\hat{\sigma}_4^2 = \frac{1}{T}\sum_{t=1}^{T}(y_t - \bar{y})^2$

≈ 0.20 need to note this is smaller than (d)



(b) $E(Y_{1960}) = \frac{1}{4}\sum_{i=1}^{4} Y^{i}_{1960} \approx \frac{1}{4}(1+2+3+4) = 2.5$

(c) $\bar{y}_4 = \frac{1}{T}\sum_{t=1920}^{2020} y_t \approx 4$

(d) $V(Y_{1960}) = \frac{1}{4}\sum_{i=1}^{4}\left[Y^{i}_{1960} - E(Y_{1960})\right]^2$

$\approx \frac{1}{4}\left[(1-2.5)^2 + (2-2.5)^2 + (3-2.5)^2 + (4-2.5)^2\right]$

$= 1.25$

3. Consider the R output listed below. With this information, answer the following (be sure to use actual values from the table in your answers):

```
model.fit = arima(data,order=c(1,0,2),method='ML')
model.fit
Coefficients:
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	ar1	ma1	ma2	intercept
	$\hat{\phi}$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{c}$
	0.0488	0.4705	0.2496	-0.0190
s.e.	0.7467	0.7376	0.3428	1.7343 $\leftarrow se(\hat{c})$

sigma2 estimated as 40.51: log likelihood = -140.76, aic = 291.53, bic = 300.33

(a) Write down the model.

(b) Is the model stationary? How do you know?

(c) What is the estimated mean of the model (i.e., μ)?

(d) Test the null hypothesis that the mean is zero.

(e) Write down the log-likelihood function for the model.

(a) $y_t = c + \phi y_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$

$\hat{y}_t = -0.0190 + 0.0488 y_{t-1} + 0.4705 \hat{\varepsilon}_{t-1} + 0.2496 \hat{\varepsilon}_{t-1}$

(b) yes $|\phi| = |0.0488| < 1$

(c) $E(y_t) = \frac{c}{1-\phi} \Rightarrow \frac{-0.0190}{1-0.0488}$

(d) $H_0: c = 0 \Rightarrow H_0: \frac{c}{1-\phi} = \mu = 0$

$t = \left| \frac{-0.0190 - 0}{1.7343} \right| < 2 \Rightarrow$ fail to reject

(e) $\ln \mathcal{L}(c, \phi, \theta_1, \theta_2, \sigma^2) = -\frac{T}{2} \ln 2\pi - \frac{T}{2} \ln \sigma^2$

$- \frac{1}{2\sigma^2} \sum_{t=1}^{T} (y_t - c - \phi y_{t-1} - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2})^2$

plug in values from table