# Economics 513: Economic Forecast and Analysis

Department of Economics, Finance and Legal Studies

University of Alabama

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Midterm I - Key

The exam consists of three questions on three pages. Each question is of equal value.

1. State (with a single sentence explanation) whether the following series are invertible or not invertible. In each case assume that $\varepsilon_t$ is a white noise sequence, $t = 1, 2, \ldots, T$. Hint: the conditions for invertibility are analogous to those for stability (i.e., stationary).

   (a) $Y_t = 0.25 + \varepsilon_t + 0.45\varepsilon_{t-1}$

   (b) $Y_t = 0.25 + \varepsilon_t + 1.45\varepsilon_{t-1}$

   (c) $Y_t = 0.25 + \varepsilon_t + 0.45\varepsilon_{t-1} + 0.35\varepsilon_{t-2}$

   (d) $Y_t = 0.25 + \varepsilon_t + 0.45\varepsilon_{t-1} + 0.05\varepsilon_{t-2} + 0.45\varepsilon_{t-3}$

   (e) $Y_t = 0.55 + 0.45Y_{t-1} + \varepsilon_t$

   (f) $Y_t = 0.25 + 1.5Y_{t-1} + \varepsilon_t$

   (g) $Y_t = 0.25 + 0.45Y_{t-1} + 0.25Y_{t-2} + \varepsilon_t$

   (h) $Y_t = 0.25 + 0.45Y_{t-1} + 0.05Y_{t-2} + 0.5Y_{t-3} + \varepsilon_t$

   (i) $Y_t = 0.25 + 0.45Y_{t-1} + 0.25Y_{t-2} + \varepsilon_t + 0.45\varepsilon_{t-1} + 0.45\varepsilon_{t-2}$

- all AR processes are invertible
- sufficient condition for invertibility $\sum_{j=1}^{q} |\theta_j| < 1$

(a) yes, $|\theta_1| < 1$

(b) no, $|\theta_1| > 1$

(c) yes, $|\theta_1| + |\theta_2| < 1$

(d) yes, $|\theta_1| + |\theta_2| + |\theta_3| < 1$

(e) yes, AR(1)

(f) yes, AR(1)

(g) yes, AR(2)

(h) yes, AR(3)

(i) yes, $|\theta_1| + |\theta_2| < 1$

2. Consider the following model: $Y_t = \mu + \varepsilon_t + \theta\varepsilon_{t-4}$ $, \varepsilon_t \sim WN$

(a) What is the common name for this model?

(b) What type of data frequency would you expect to form this type of model.

(c) Derive the expected value of the series.

(d) Derive the variance of the series.

(e) Derive the autocovariance of the series for all lags $j = 1, 2, \ldots$.

(f) Derive the autocorrelation for all all lags $j = 1, 2, \ldots$. Plot the autocorrelation function.

(g) State the condition for which the model is invertible.

(h) Assuming that the model is invertible, write it as an $AR(\infty)$.

(a) $ARMA(0,(0,4))$ or $ARMA(0,4)$

(b) quarterly data

(c) $E(Y_t) = E(\mu + \varepsilon_t + \theta\varepsilon_{t-4}) = \mu$

(d) $\gamma_0 = E[(Y_t-\mu)^2] = E[(\varepsilon_t + \theta\varepsilon_{t-4})^2]$

$= \sigma^2(1+\theta^2)$

(e) $\gamma_4 = E[(Y_t-\mu)(Y_{t-4}-\mu)]$

$= E[(\varepsilon_t + \theta\varepsilon_{t-4})(\varepsilon_{t-4} + \theta\varepsilon_{t-8})] = \theta\sigma^2$

$\gamma_j = 0 \;\; \forall j \neq 4, 0$

(f) $\rho_j = \gamma_j/\gamma_0$, $\rho_4 = \frac{\gamma_4}{\gamma_0} = \frac{\theta}{1+\theta^2}$

$\rho_j = 0 \;\; \forall j \neq 4$



(g) $|\theta| < 1$

$AR(\infty)$

(h) $Y_t = \mu + \varepsilon_t(1+\theta L) \Rightarrow \frac{Y_t}{(1+\theta L)} = \frac{\mu}{1+\theta} + \varepsilon_t$

3. Consider the R output listed below. With this information, answer the following:

model.fit = arima(data,order=c(1,0,0),method='ML')
model.fit
Coefficients:

| | ar1 | intercept |
|---|---|---|
| | 0.4796 | 179.4921 |
| s.e. | 0.0565 | 0.4268 |

sigma2 estimated as 6.495: log likelihood = -126.24, aic = 296.48

(a) What is the common name for this model? Write down this model.

(b) What estimation method is used to estimate this model? Write down that objective function.

(c) Test the null that this series is mean zero (write down the null and alternative hypothesis, test statistic, and decision rule).

(d) Test the null that this series is only a function of white noise sequences (write down the null and alternative hypothesis, test statistic, and decision rule).

(e) Suppose we believed these estimates to be the true parameters, draw the autocorrelation function and partial autocorrelation function.

(a) AR(1) : $y_t = c + \phi y_{t-1} + \varepsilon_t$

(b) maximum likelihood estimation

$$\mathcal{L} = -\frac{T}{2}\ln 2\pi - \frac{T}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum_{t=1}^{T}(y_t - \hat{c} - \hat{\phi} y_{t-1})^2$$

(c) $H_0: \phi = 0$, $H_1: \phi \neq 0$

$t_{\frac{\alpha}{2},T-1}$

$$t = \frac{\hat{\phi} - 0}{se(\hat{\phi})} = \frac{0.4796}{0.0565} > 2 \Rightarrow \text{reject}$$

(d) $H_0: c = 0$, $H_1: c \neq 0$

$t_{\frac{\alpha}{2},T-1}$

$$t = \frac{\hat{c} - 0}{se(\hat{c})} = \frac{179.4921}{0.4268} > 2 \Rightarrow \text{reject}$$

(e)