Economics 513: Economic Forecast and Analysis

Department of Economics, Finance and Legal Studies

University of Alabama

Spring 2022

Final Exam — Key

The exam consists of four questions on four pages. Each question is of equal value.

1. For each model below (for $t = 1, 2, \ldots, t', \ldots, T$), determine whether the sequence Y_t is stationary or nonstationary. For each case, $\varepsilon_t \sim WN$ (a white noise sequence). Show your work.

   (a) $Y_t = \mu + \varepsilon_t$

   (b) $Y_t = \mu + \varepsilon_t + \theta\varepsilon_{t-1}$, where $\theta = 1$

   (c) $Y_t = \alpha + \beta t + \varepsilon_t$

   (d) $Y_t = \alpha + \delta D_t + \varepsilon_t$, where $D_t = 1$ if $t \geq t'$ and 0 otherwise

   (e) $Y_t = \alpha + \beta Y_{t-1} + \varepsilon_t$, where $\alpha = 0$ and $\beta = 1$

(a) $E(Y_t) = E(\mu + \varepsilon_t) = \mu \quad \forall t$

$$\gamma_{jt} = E[(Y_t - \mu)(Y_{t-j} - \mu)] = E(\varepsilon_t \varepsilon_{t-j})$$

$$= \begin{cases} \sigma^2 & \text{for } j = 0 \\ 0 & \text{for } j \neq 0 \end{cases}$$

$\Rightarrow$ stationary

(b) $E(Y_t) = E(\mu + \varepsilon_t + \varepsilon_{t-1}) = \mu \quad \forall t$

$$\gamma_{jt} = E[(Y_t - \mu)(Y_{t-j} - \mu)] = E[(\varepsilon_t + \varepsilon_{t-1})(\varepsilon_{t-j} + \varepsilon_{t-j-1})]$$

$$= \begin{cases} 2\sigma^2 & \text{for } j = 0 \\ \sigma^2 & \text{for } j = 1 \\ 0 & \text{for } j > 1 \end{cases} \quad \forall t$$

$\Rightarrow$ stationary

(c) $E(Y_t) = E(\alpha + \beta t + \varepsilon_t) = \alpha + \beta t$

$\Rightarrow$ nonstationary

(d) $E(Y_t) = E(\alpha + \delta D_t + \varepsilon_t)$

$$= \begin{cases} \alpha + \delta & \text{if } t \geq t' \\ \alpha & \text{if } t < t' \end{cases}$$

$\Rightarrow$ nonstationary

(e) $Y_t = Y_{t-1} + \varepsilon_t$

$= Y_0 + \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_t$

$E(Y_t) = E(Y_0 + \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_t) = Y_0 \quad \forall t$

$\gamma_{0t} = E[(Y_t - \mu)^2] = E[(\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_t)^2]$

$= t\sigma^2$

$\Rightarrow$ nonstationary

2. In practice, the theoretical mean (μ), variance (γ_0), autocovariance (γ_j) and autocorrelation (ρ_j) are unknown to the researcher. Suppose we have a sample of data $\{y_t\}_{t=1}^{T}$, and assume that the series is stationary. With this information, answer the following:

(a) Give an estimator for μ.

(b) Give an estimator for γ_0.

(c) Give an estimator for γ_j.

(d) What equation do we use for the sample ACF to estimate ρ_j?

(e) Using the equation from part (d), write down the estimator of ρ_j

(a) $\hat{\mu} = \bar{y} = \frac{1}{T} \sum_{t=1}^{T} y_t$

(b) $\hat{\gamma}_0 = \frac{1}{T-1} \sum_{t=1}^{T} (y_t - \bar{y})^2$

(c) $\hat{\gamma}_j = \frac{1}{T-j-1} \sum_{t=j+1}^{T} (y_t - \bar{y})(y_{t-j} - \bar{y})$

(d) $y_t = \alpha + \rho_j y_{t-j} + u_t$

(e) $$\hat{\rho}_j = \frac{\sum_{t=j+1}^{T} (y_t - \bar{y})(y_{t-j} - \bar{y})}{\sum_{t=j+1}^{T} (y_{t-j} - \bar{y})^2}$$

3. Consider a random sample of data $\{y_t\}_{t=1}^{T}$ and the model $y_t = c + \phi y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$, where $|\phi| < 1$, $|\theta| < 1$ and $\varepsilon_t \sim N(0, \sigma^2)$ is a white noise sequence. With this information, answer the following:

(a) Write the log-likelihood function for this model.

(b) What is the in-sample forecast for y_T?

(c) What is the out-of-sample forecast for y_{T+1}?

(d) What is the out-of-sample forecast interval for y_{T+1}?

(e) What is the out-of-sample density forecast for y_{T+1}?

(a) $$\ln \mathcal{L}(c, \phi, \theta, \sigma^2) = -\frac{(T-1)}{2}\ln 2\pi - \frac{(T-1)}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum_{t=2}^{T}(y_t - c - \phi y_{t-1} - \theta\varepsilon_{t-1})^2$$

(b) $$\hat{y}_T = \hat{c} + \hat{\phi} y_{T-1} + \hat{\theta}\hat{\varepsilon}_{T-1}$$

(c) $$\hat{y}_{T+1|T} = \hat{c} + \hat{\phi} y_T + \hat{\theta}\hat{\varepsilon}_T$$

(d) $$e_{T+1} = y_{T+1} - \hat{y}_{T+1|T} = \varepsilon_{T+1}$$

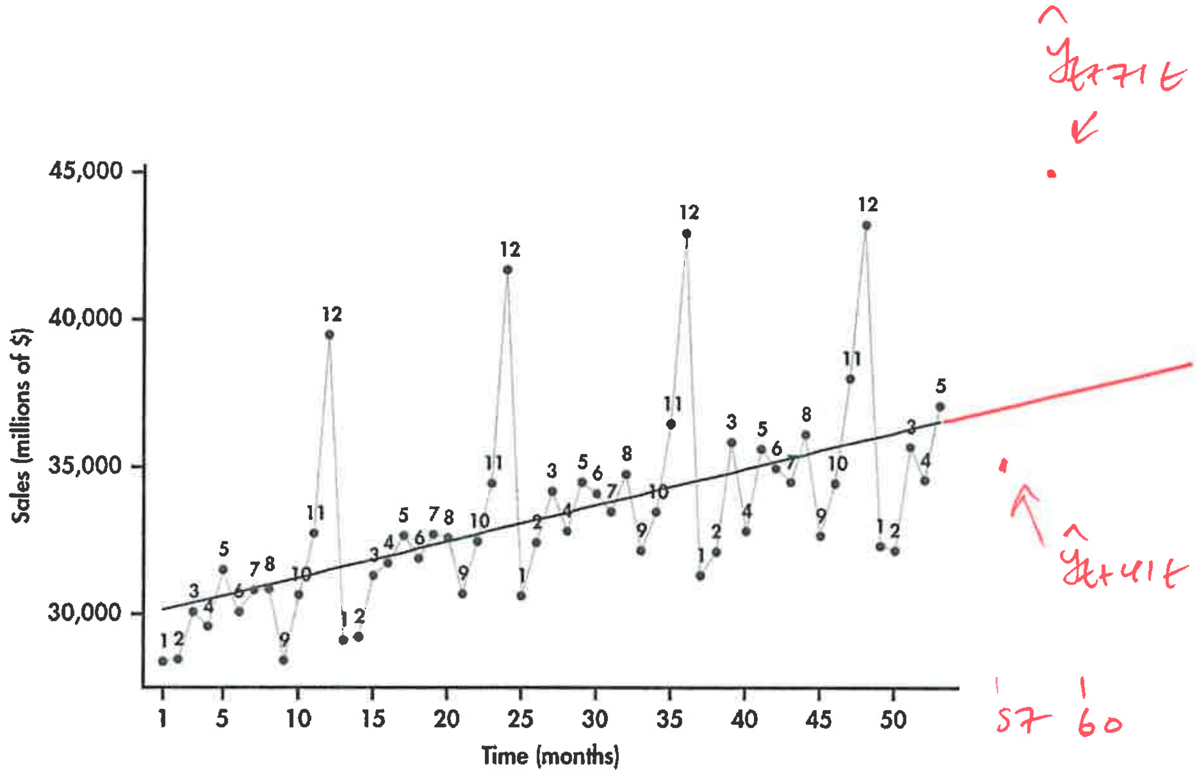
$$V(e_{T+1}) = V(\varepsilon_{T+1}) = \sigma^2$$

assuming $\varepsilon_t \sim N(0, \sigma^2)$

$$[\hat{y}_{T+1|T} \pm 1.96\,\sigma]$$

(e) $$N(\hat{y}_{T+1|T}, \sigma^2)$$

assuming $\varepsilon_t \sim N(0, \sigma^2)$



4. The figure above is a time series for monthly warehouse club and superstore sales from January 2010 through May 2014. With this information, answer the following:

   (a) Propose a tenative model for this non-stationary series.

   (b) Using the model you propose in part (a), what is the out-of-sample point forecast for 4 periods ahead (September 2014), $\widehat{y}_{t+h|t}$. Draw a reasonable value for this forecast on the figure.

   (c) Using the model you propose in part (a), what is the out-of-sample point forecast for 7 periods ahead (December 2014), $\widehat{y}_{t+h|t}$. Draw a reasonable value for this forecast on the figure.

   (d) For the model you propose in part (a), how would you make the series stationary?

   (e) Using the stationary version of the series from part (d), draw an analogous figure to that above.

(a) $y_t = \alpha + \beta t + \phi_{12} y_{t-12} + \varepsilon_t$

(b) $y_{t+4} = \alpha + \beta(t+4) + \phi_{12} y_{t+4-12} + \varepsilon_{t+4}$

$\hat{y}_{t+4|t} = \alpha + \beta(t+4) + \phi_{12} y_{t+4-12}$

(c) $y_{t+7} = \alpha + \beta(t+7) + \phi_{12} y_{t+7-12} + \varepsilon_{t+7}$

$\hat{y}_{t+7|t} = \alpha + \beta(t+7) + \phi_{12} y_{t+7-12}$

(d) $y_t = a + bt + z_t$

$\hat{z}_t = \cancel{y_t} - \hat{a} - \hat{b}t$

$\hat{z}_t = \phi_{12} y_{t-12} + \varepsilon_t$

(c)