Economics 471: Econometrics

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Problem Set #1 – Answers

1. (a) Average years of education and salary are 13.75 and 44.92, respectively. Given that there are an even number of observations, the median are given by the average of the 6th and 7th order statistics. Thus the median years of education is 13 and the median salary is 40.
   (b) See figure 1. It looks roughly linear.
   (c) See figure 2
   (d) The covariance is 114.90 and the correlation coefficient is 0.909471.
   (e) The covariance is 1148.96 and the correlation coefficient is 0.909471.
   (f) The correlation coefficient does not depend on the units being measured. It is generally considered as more reliable.

2. $\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})=\sum_{i=1}^{n}x_i(y_i-\overline{y})-\sum_{i=1}^{n}\overline{x}(y_i-\overline{y})=\sum_{i=1}^{n}x_i(y_i-\overline{y})-\overline{x}\sum_{i=1}^{n}(y_i-\overline{y})=\sum_{i=1}^{n}x_i(y_i-\overline{y})-\overline{x}*0=\sum_{i=1}^{n}x_i(y_i-\overline{y})$.Similarly, $\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})=\sum_{i=1}^{n}y_i(x_i-\overline{x})-\sum_{i=1}^{n}\overline{y}(x_i-\overline{x})=\sum_{i=1}^{n}y_i(x_i-\overline{x})-\overline{y}\sum_{i=1}^{n}(x_i-\overline{x})=\sum_{i=1}^{n}y_i(x_i-\overline{x})-\overline{y}*0=\sum_{i=1}^{n}y_i(x_i-\overline{x})$.
   (a) See figure 3
   (b) Marginal product is the change in cheatsheets for an additional hour work (the partial derivative). $MP=\frac{\partial cheatsheets}{\partial hours}=\left(\frac{1}{2}\right)2*hours^{-1/2}=1/\sqrt{hours}$.
   (c) Assume that it takes the linear form $\#cheatsheets=1+2*hours$. The marginal product would be 2. This is constant whereas the other diminishes over time.
   (d) In economics we generally assume a diminishing marginal product. Given that you are the sole person writing the cheatsheets you will probably get tired as time goes on and thus be less productive.

   (a) Discrete
   (b)

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

   Probability, $p(X=0)=6/36, p(X=1)=10/36, p(X=2)=8/36, p(X=3)=6/36, p(X=4)=$

$4/36$, and $p(X=5)=2/36$. Cummulative probability $p(X\le 0)=6/36$, $p(X\le 1)=16/36$, $p(X\le 2)=24/36$, $p(X\le 3)=30/36$, $p(X\le 4)=34/36$, and $p(X\le 5)=36/36$.

(c)

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

Probability, $p(X=1)=1/36$, $p(X=2)=3/36$, $p(X=3)=5/36$, $p(X=4)=7/36$, $p(X=5)=9/36$ and $p(X=6)=11/36$. Cummulative probability $p(X\le 1)=1/36$, $p(X\le 2)=4/36$, $p(X\le 3)=9/36$, $p(X\le 4)=16/36$, $p(X\le 5)=25/36$ and $p(X\le 6)=36/36$

(d) The expected value of X in (a) and (c) is 1.94 and 4.47, respectively..

3. Show that $V(X)=E\left(X^2\right)-[E(X)]^2$.

4. The following table gives the college GPA of students versus their corresponding high school GPA

		High School GPA			
		2.0	3.0	4.0	Total
College GPA	2.0	12	3	3	18
	3.0	6	9	6	21
	4.0	2	21	7	30
	Total	20	33	16	69

(a)

		High School GPA			
		2.0	3.0	4.0	Total
College GPA	2.0	12/69	3/69	3/69	18
	3.0	6/69	9/69	6/69	21
	4.0	2/69	21/69	7/69	30
	Total	20	33	16	69

(b)

		High School GPA			
		2.0	3.0	4.0	Total
College GPA	2.0	12/20	3/33	3/16	18
	3.0	6/20	9/33	6/16	21
	4.0	2/20	21/33	7/16	30
	Total	20	33	16	69

(c) $E(College\ GPA \mid High\ School\ GPA=2.0)=2.5$, $E(College\ GPA \mid High\ School\ GPA=3.0)=3.55$, $E(College\ GPA \mid High\ School\ GPA=4.0)=3.25$.

(d) See figure 4.

(e) The conditional expectations do agree with a priori expectations about the relationship between high school and college performance. It seems that those who perform better in high school also perform better in college. Those who received a 4.0 in high school will have a 44% better chance of mainintaing this GPA in college with a 33% chance of getting a 3.0. These results are on the whole not surprising with the exception that those who receive a 3.0 in college have an even higher chance of getting a 4.0 in college.

5. Unbiasedness is neither necessary nor sufficient for consistency of an estimator. We can have an inconsistent and unbiased estimator, an inconsistent and biased estimator, a consistent and unbiased estimator, or a connsistent and biased estimator. See the lecture notes for examples of each.

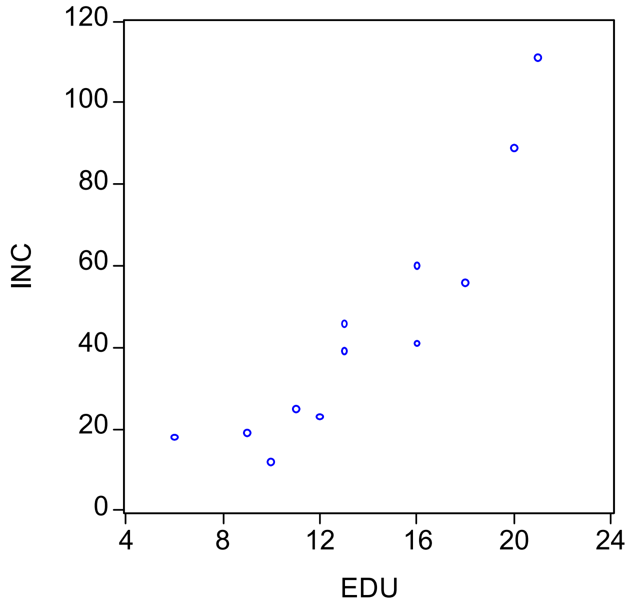

Figure 1:

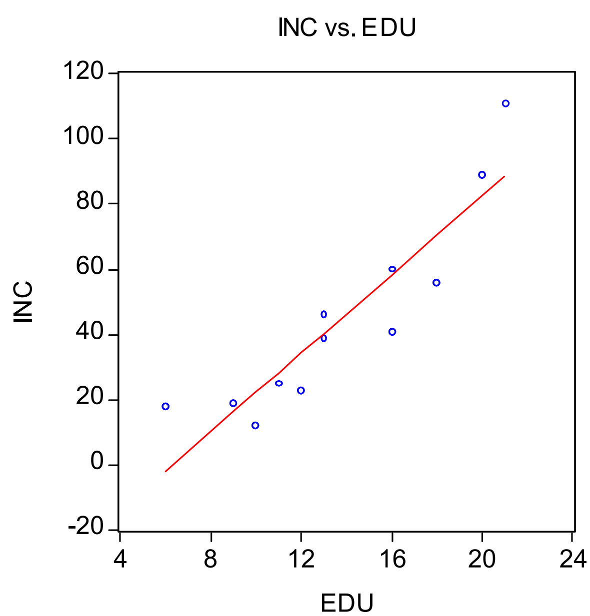

Figure 2:

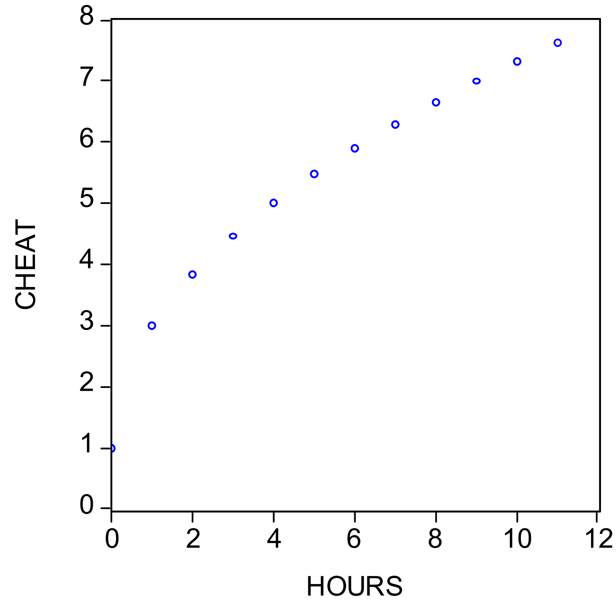

Figure 3:

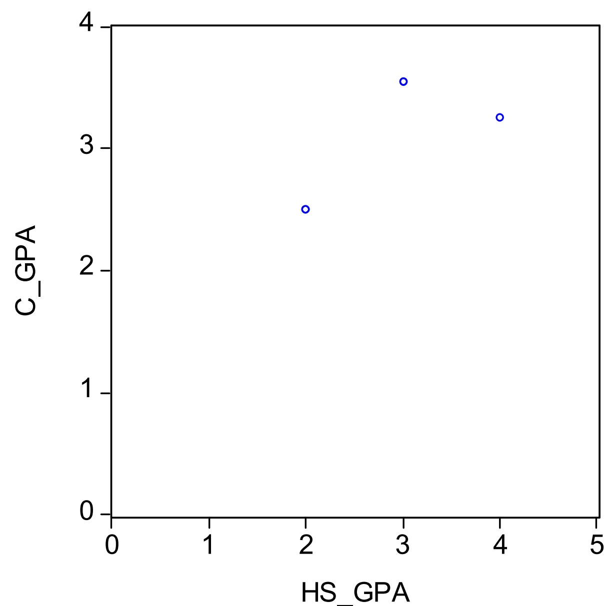

Figure 4: