Economics 471: Econometrics

Department of Economics, Finance and Legal Studies
University of Alabama
Fall 2014

Midterm II – Answers

1. (a) Take the natural logarithm of each side: $\ln y = \ln\alpha + \beta_1 \ln x_1 + \beta_2 \ln x_2 + u$.

   (b) $H_0 : \beta_1 + \beta_2 = 1$ vs. $H_1 : \beta_1 + \beta_2 \neq 1$. $t = \frac{\widehat{\beta}_1+\widehat{\beta}_2-1}{se\left(\widehat{\beta}_1+\widehat{\beta}_2\right)}$.

   (c) $\beta_1 + \beta_2 = 1 \Rightarrow \beta_1 = 1 - \beta_2 \Rightarrow \ln y = \ln\alpha + (1-\beta_2)\ln x_1 + \beta_2 \ln x_2 + u \Rightarrow \ln y = \ln\alpha + \ln x_1 + \beta_2(\ln x_1 - \ln x_2) + u \Rightarrow \ln y - \ln x_1 = \ln A + \beta(\ln L - \ln K) + u$.

   (d) $H_0 : \beta_1 = \beta_2 = 0$ vs. $H_1 : H_0$ is not true. $F = \frac{(SSR_r - SSR_u)/q}{SSR_r/(n-k-1)} = \frac{\left(R_u^2 - R_r^2\right)/q}{(1-R_u^2)/n-k-1} = \frac{R^2/q}{(1-R^2)/(n-k-1)}$, where $q = k = 2$ and $n-k-1 = n-2-1$. Note that the second R^2 is only equivalent when testing for overall significance of the regression.

   (e) Under the null $\beta_1 = \beta_2 = 0 \Rightarrow \ln y = \ln\alpha + u$ Therefore $\sum_{i=1}^n \widehat{u}_i^2 = \sum_{i=1}^n \left(y_i - \widehat{\ln\alpha}\right)^2$. $\partial \sum_{i=1}^n \widehat{u}_i^2 / \partial\widehat{\ln\alpha} = -2\sum_{i=1}^n \left(y_i - \widehat{\ln\alpha}\right) = 0$. Solving for $\widehat{\ln\alpha}$ we obtain $\sum_{i=1}^n \left(y_i - \widehat{\ln\alpha}\right) = 0 \Rightarrow \widehat{\ln\alpha} = 1/n\sum_{i=1}^n y_i = \overline{y}$.

   (f) Under the null $\beta_1 = \beta_2 = 0 \Rightarrow \widehat{\ln\alpha} = 1/n\sum_{i=1}^n y_i$. Therefore $V\left(\widehat{\ln\alpha}\right) = V\left(1/n\sum_{i=1}^n y_i\right) = 1/n^2\sum_{i=1}^n V(y_i) = \sigma^2/n$.

2. (a) $\sum_{i=1}^{n} \widehat{\eta}_i = \sum_{i=1}^{n} y_i - \widehat{\heartsuit} - \widehat{\diamondsuit} x_i - \widehat{\maltese} z_i = \sum_{i=1}^{n} y_i - \left(\overline{y} - \widehat{\diamondsuit}\overline{x} - \widehat{\maltese}\overline{z}\right) - \widehat{\diamondsuit} x_i - \widehat{\maltese} z_i = n\overline{y} - n\overline{y} + \widehat{\diamondsuit} n\overline{x} - \widehat{\diamondsuit} n\overline{x} + \widehat{\maltese} n\overline{z} - \widehat{\maltese} n\overline{z} = 0$

   (b) $\sum_{i=1}^{n} \widehat{\eta}_i z_i = \sum_{i=1}^{n} \left(y_i - \widehat{\heartsuit} - \widehat{\maltese} z_i\right) z_i = \sum_{i=1}^{n} \left[y_i - \left(\overline{y} - \widehat{\maltese}\overline{z}\right) - \widehat{\maltese} z_i\right] z_i = \sum_{i=1}^{n} \left[(y_i - \overline{y}) - \widehat{\maltese}(z_i - \overline{z})\right] z_i = \sum_{i=1}^{n} \left[(y_i - \overline{y}) - \widehat{\maltese}(z_i - \overline{z})\right](z_i - \overline{z}) = \sum_{i=1}^{n} \left[(y_i - \overline{y})(z_i - \overline{z}) - \widehat{\maltese}(z_i - \overline{z})^2\right] = \sum_{i=1}^{n} (y_i - \overline{y})(z_i - \overline{z}) - \widehat{\maltese}\sum_{i=1}^{n} (z_i - \overline{z})^2 = \sum_{i=1}^{n} (y_i - \overline{y})(z_i - \overline{z}) - \frac{\sum_{i=1}^{n}(y_i-\overline{y})(z_i-\overline{z})}{\sum_{i=1}^{n}(z_i-\overline{z})^2}\sum_{i=1}^{n} (z_i - \overline{z})^2 = \sum_{i=1}^{n} (y_i - \overline{y})(z_i - \overline{z}) - \sum_{i=1}^{n} (y_i - \overline{y})(z_i - \overline{z}) = 0$

3. (a) Table 1: 0.005334, Table 2: $-0.006713 + 0.005586 * PRIGPA$

   (b) Table 1: 0.402373, Table 2: $-1.628540 + 2 * 0.295905 * PRIGPA + 0.005586 * ATNDRTE$

   (c) Table 1: 0.084257, Table 2: $-0.128039 + 2 * 0.004533 * ACT$

   (d) Table 1: $F = \frac{\left(R_u^2 - R_r^2\right)/q}{(1-R_u^2)/(n-k-1)} = \frac{(0.201308-0)/3}{(1-0.201308)/(680-3-1)} = 56.79461252 > F_{3,674,0.05} \approx F_{3,120,0.05} = 2.68$, Table 2: $F = \frac{\left(R_u^2 - R_r^2\right)/q}{(1-R_u^2)/(n-k-1)} = \frac{(0.228654-0)/6}{(1-0.228654)/(680-6-1)} = 33.25013288 > F_{6,673,0.05} \approx F_{6,120,0.05} = 2.17$

(e) $F = \frac{(SSR_R - SSR_u)/q}{SSR_u/(n-k-1)} = \frac{(530.9411 - 512.7624)/3}{512.7624/(680-6-1)} = \frac{\left(R_u^2 - R_r^2\right)/q}{(1-R_u^2)/(n-k-1)} = \frac{(0.228654 - 0.201308)/3}{(1-0.228654)/(680-6-1)} = 7.953135601 > F_{3,673,0.05} \approx F_{3,120,0.05} = 2.68$