Economics 471: Econometrics

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Midterm II – Answers

1. (a) i. The model is correctly specified, $y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \cdots \beta_k x_k + u$
   ii. The error is random, $E(u) = 0$
   iii. The error is uncorrelated with the regressor, $E(u|x_1, x_2, \ldots, x_k) = 0$
   iv. No perfect collinearity
   v. The error is homoskedastic, $V(u|x_1, x_2, \ldots, x_k) = \sigma^2$
   vi. The error is independent of $x_1, x_2, \ldots, x_k$ and is distributed $N(0, \sigma^2)$

   (b) The first four assumptions guarantee that OLS is unbiased

   (c) The first five assumptions guarantee that OLS is BLUE (best linear unbiased estimator)

   (d) The first five assumptions guarantee that OLS is MVUE (minimum variance unbiased estimator). Note that assumption six implies 2, 3 and 5

2. (a) ♣ is the overall intercept of the regression model. ♠ $= \frac{\partial y}{\partial x}$, the partial effect of a one unit change in x on y. ▼, ★ and ■ each represent linear shifts in the (intercept of the) regression model for groups *married_male*, *married_female*, and *single_female*, respectively, with respect to the base category *single_male*.

   (b) married men: ♣ + ▼, married women: ♣ + ★, single women: ♣ + ■ and single men: ♣

   (c) This is a violation of Gauss-Markov assumption four: perfect collinearity. The model will not run as one variable is a perfect linear combination of one or more other variables.

   (d) The fact that they are all positive says that, holding x constant, each of the groups has a expected wage larger than single males. The ordering shows that, holding x constant, married men have higher expected wages than married females who have higher expected wages than single females.

   (e) Given the information we know from part (d), single women are the second highest expected wage group for a fixed level of x. Therefore we expect the coefficient on married men (▼) to remain positive, while the coefficients for single women (■) and single men (♦) will be negative.

3. (a) In the linear model, the partial effect $\frac{\partial y}{\partial x} = \beta$, is the coefficient on x. In the quadratic model, the partial effect $\frac{\partial y}{\partial x} = \beta + 2\gamma x$ is a function of x. Plugging in the values from the table we see that $\frac{\partial \widehat{y}}{\partial x} = 4.0113$ in the linear model and $\frac{\partial \widehat{y}}{\partial x} = 9.5326 + 2(-1.6915)x = 9.5326 - 3.3832x$.

   (b) In the linear model a one-unit increase in homework will bring about the same change in test score regardless of the value of homework. Given that we know the maximum score is 100, we need to find the value of homework that gives the predicted test score equal to 100. 100 =

$49.838 + 4.0113 * homework^* \Rightarrow homework^* = (100 - 49.838)/4.0113 = 12.50$ (insane) hours per day.

(c) For the quadratic model with $\beta > 0$ and $\gamma < 0$ we know that the function achieves its maximum when $\frac{\partial y}{\partial x} = \beta + 2\gamma x = 0 \Rightarrow 9.5326 - 3.3832 * homework^* = 0 \Rightarrow homework^* = 9.5326/3.3832 = 2.8176 \Rightarrow$ the maximum test score is equal to $47.232 + 9.5326 * 2.8176 - 1.6916 * 2.8176^2 = 60.6616$.

(d) The easiest way to do this is to test the null: $H_0 : \gamma = 0$ in the quadratic model. The t-statistic for this test is -9.0473 and its respective $p - value = 0.0000$. Therefore we reject the model is linear. A more complicated test would be to take the RSS from both the linear and quadratic models, construct the F-statistic$= \frac{(332301.3-325165.7)/1}{325165.7/(3733-2-1)} = 81.835$ and compare them to the critical value from the F-table$= 3.84$ which would also suggest to reject the null. Note that $t^2 = F$ in this case of a single restriction.

(e) In the linear model: a test for the validity of the regression is the null: $H_0 : \beta = 0$. Note that the F-statistic is not avaiable in this EViews output and thus we must use the R^2 version of the test, noting that the resticted model which has no regressors will have a $R^2 = 0$. $F = \frac{R^2/q}{(1-R^2)/(n-k-1)} = \frac{0.0271/1}{(1-0.0271)/(3733-1-1)} = 103.9265 > 3.84$ and thus we reject the null. An alternative would be to note that this is a single restrict and $t^2 = 10.19134^2 = 103.9265$. For the quadratic model $F = \frac{R^2/q}{(1-R^2)/(n-k-1)} = \frac{0.0479/2}{(1-0.0479)/(3733-2-1)} = 93.8279 > 3.00$ (critical value) and thus we reject the null. No t alternative is possible in this case.

(f) Here we list all six selection criteria available from the EViews output. Note that the quadratic model performs better in each. It has a higher value for each of the first two selection criteria and a smaller value for the remaining selection criteria.

	Selection Criteria				
	R^2	$\overline{R}^2$	$\widehat{\sigma}$	AIC	SC
linear model	0.0271	0.0268	9.4374	7.3278	7.3311
quadratic model	0.0479	0.0475	9.3368	7.3066	7.3116