Economics 471: Introductory Econometrics

Department of Economics, Finance and Legal Studies

University of Alabama

Fall 2022

Midterm II - Key

The exam consists of three questions on four pages. Each question is of equal value.

1. Consider a random sample of data $\{x_i, y_i\}_{i=1}^n$ and the model $y_i = \alpha + \beta x_i + u_i$, where $E(u_i|x_i) = 0$ and $V(u_i|x_i) = \sigma^2$. Consider the estimator of β

$$\widetilde{\beta} = \frac{\sum_{i=1}^n \widehat{r}_i y_i}{\sum_{i=1}^n \widehat{r}_i^2},$$

where $x_i = \gamma_0 + r_i$. With this information, answer the following questions:

   (a) Derive the least-squares estimator of the intercept parameter α.

   (b) What model is used to obtain $\widehat{r}_i$?

   (c) Derive the least-squares estimator of the intercept parameter γ_0.

   (d) Using the result from part (c), show that $\widetilde{\beta}$ is equivalent to the slope estimator we derived in class (i.e., $\widehat{\beta} = \sum_{i=1}^n (y_i - \bar{y})(x_i - \bar{x}) / \sum_{i=1}^n (x_i - \bar{x})^2$).

   (e) Noting your result from part (d), what is the conditional variance of $\widetilde{\beta}$ (i.e., $Var(\widetilde{\beta}|x)$)?

(a) $\sum_{i=1}^n \hat{u}_i^2 = \sum_{i=1}^n (y_i - \hat{\alpha} - \hat{\beta} x_i)^2$

$\frac{\partial}{\partial \alpha} = -2 \sum_{i=1}^n (y_i - \hat{\alpha} - \hat{\beta} x_i) = 0 \Rightarrow$

$\sum_{i=1}^n y_i - \sum_{i=1}^n \hat{\alpha} - \hat{\beta} \sum_{i=1}^n x_i = 0 \Rightarrow$

$n\hat{\alpha} = \sum_{i=1}^n y_i - \hat{\beta} \sum_{i=1}^n x_i = 0$

$= \bar{y} - \hat{\beta}\bar{x} = 0$

(b) $x_i = \gamma_0 + r_i \Rightarrow \hat{r}_i = x_i - \hat{\gamma}_0$

(c) $\sum_{i=1}^n \hat{r}_i^2 = \sum_{i=1}^n (x_i - \hat{\gamma}_0)^2$

$\frac{\partial}{\partial \gamma_0} = -2 \sum_{i=1}^n (x_i - \hat{\gamma}_0) = 0 \Rightarrow \sum_{i=1}^n x_i = \sum_{i=1}^n \hat{\gamma}_0 \Rightarrow$

$n\hat{\gamma}_0 = \sum_{i=1}^n x_i \Rightarrow \hat{\gamma}_0 = \frac{1}{n}\sum_{i=1}^n x_i = \bar{x}$

(d)

$$\tilde{\beta} = \frac{\sum_{i=1}^{n} \hat{r}_i y_i}{\sum_{i=1}^{n} \hat{r}_i^2} = \frac{\sum_{i=1}^{n} (x_i - \hat{x}_i) y_i}{\sum_{i=1}^{n} (x_i - \hat{x}_i)^2}$$

$$= \frac{\sum_{i=1}^{n} (x_i - \bar{x}) y_i}{\sum_{i=1}^{n} (x_i - \bar{x})^2} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} = \hat{\beta}$$

(e) since $\tilde{\beta} = \hat{\beta} \Rightarrow V(\tilde{\beta} | x) = V(\hat{\beta} | x)$

$$= \frac{\sigma^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

2. Consider a random sample of data $\{x_{1i}, x_{2i}, y_i\}_{i=1}^{n}$ and the model $y_i = \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + u_i$, where $E(u_i|x_{1i}, x_{2i}) = 0$. With this information, answer the following:

(a) Derive the method of moments estimator of the intercept parameter α.

(b) For the null $H_0 : \beta_1 = 0$, give the test statistic and the distribution of the test statistic under the null hypothesis.

(c) Consider the test discussed in part (b), draw the null distribution and indicate the rejection region.

(d) For the null $H_0 : \beta_1 = \beta_2 = 0$, give the test statistic and the distribution of the test statistic under the null hypothesis.

(e) Consider the test discussed in part (d), draw the null distribution and indicate the rejection region.

(a) $E(u) = 0 \Rightarrow \frac{1}{n}\sum_{i=1}^{n} u_i = 0 \Rightarrow$

$$\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{\alpha} - \hat{\beta}_1 x_{1i} - \hat{\beta}_2 x_{2i}) = 0$$

$$\bar{y} - \hat{\alpha} - \hat{\beta}_1 \bar{x}_1 - \hat{\beta}_2 \bar{x}_2 = 0$$

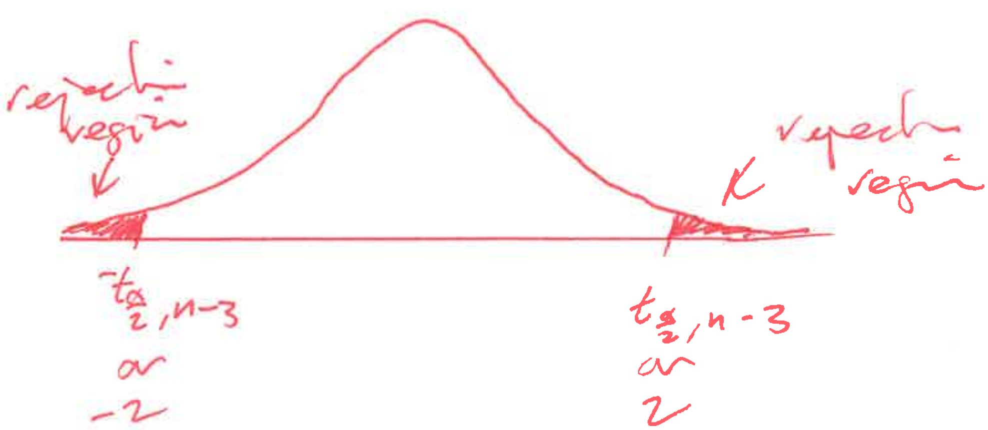
$$\hat{\alpha} = \bar{y} - \hat{\beta}_1 \bar{x}_1 - \hat{\beta}_2 \bar{x}_2$$

(b) $H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$

$$t = \frac{\hat{\beta}_1 - 0}{se(\hat{\beta}_1)} \sim t_{n-3}$$

(c)



(d) $H_0: \beta_1 = \beta_2 = 0$

H_A: H_0 is not true

restricted

$y = \alpha + u$

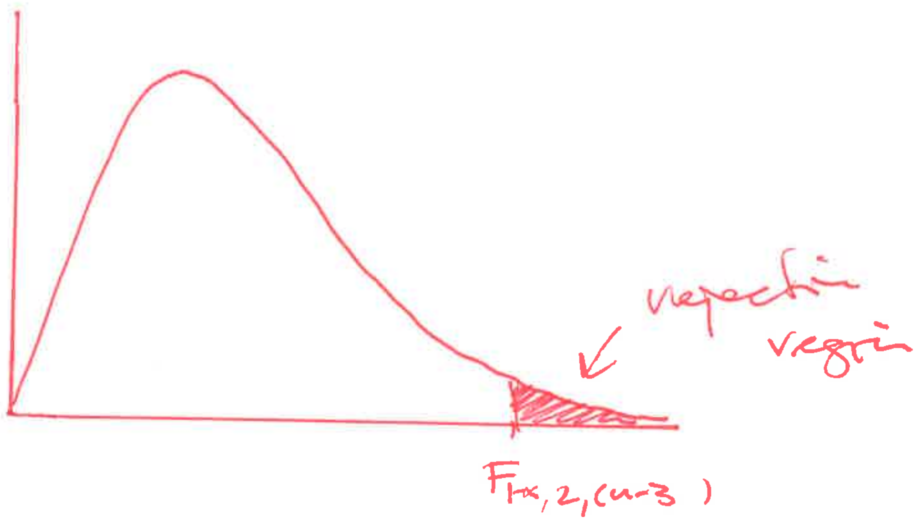
unrestricted

$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + u$

$$F = \frac{(SSR_R - SSR_u)/2}{SSR_u/(n-3)} \sim F_{2,(n-3)}$$

R^2 formula is also ok

(e)



3. Consider the two pieces of gretl output below which relates semester GPA (*termgpa*) to the previous semesters GPA (*priGPA*), ACT score (*ACT*), class attendance rate (*atndrte*), homework completion rate (*hwrte*), and in Model 2, the squares of *atndrte* (*sqatndrte*) and *hwrte* (*sqhwrte*) as well as the interaction of *atndrte* and *hwrte* (*atndrtehwrte*). Using this information, answer the following (be specific):

Model 1: OLS, using observations 1–680 (n = 674)
Missing or incomplete observations dropped: 6
Dependent variable: termgpa

	Coefficient	Std. Error	t-ratio	p-value
const	−1.28748	0.165892	−7.761	0.0000
priGPA	0.557014	0.0423614	13.15	0.0000
ACT	0.0358472	0.00604693	5.928	0.0000
atndrte	0.0101788	0.00155644	6.540	0.0000
hwrte	0.00928213	0.00122704	7.565	0.0000

Mean dependent var	2.613838	S.D. dependent var	0.725003
Sum squared resid	152.2166	S.E. of regression	0.477000
R^2	0.569704	Adjusted R^2	0.567131
$F(4, 669)$	221.4360	P-value(F)	6.0e–121
Log-likelihood	−454.9336	Akaike criterion	919.8672
Schwarz criterion	942.4334	Hannan–Quinn	928.6056

Model 2: OLS, using observations 1–680 (n = 674)
Missing or incomplete observations dropped: 6
Dependent variable: termgpa

	Coefficient	Std. Error	t-ratio	p-value
const	−1.26265	0.295228	−4.277	0.0000
priGPA	0.551005	0.0426538	12.92	0.0000
ACT	0.0358668	0.00604662	5.932	0.0000
atndrte	0.00486132	0.00653044	0.7444	0.4569
sqatndrte	0.000108157	6.43411e–005	1.681	0.0932
hwrte	0.0121354	0.00570628	2.127	0.0338
sqhwrte	4.61294e–005	4.81473e–005	0.9581	0.3384
atndrtehwrte	−0.000128909	8.08380e–005	−1.595	0.1113

Mean dependent var	2.613838	S.D. dependent var	0.725003
Sum squared resid	151.4907	S.E. of regression	0.476931
R^2	0.571756	Adjusted R^2	0.567255
$F(7, 666)$	127.0269	P-value(F)	3.5e–118
Log-likelihood	−453.3228	Akaike criterion	922.6455
Schwarz criterion	958.7514	Hannan–Quinn	936.6269

(a) What is the marginal impact of a 1% increase in the attendance rate (*atndrte*) on semester GPA (*termgpa*) in Model 1? In Model 2?

(b) Test the null hypothesis that the square terms (i.e., *sqatndrte* and *sqhwrte*) and interaction term (*atndrtehwrte*) are jointly irrelevant in the prediction of semester GPA (*termgpa*).

(c) Test the null hypothesis that all of the regressors (*priGPA*, *ACT*, *atndrte* and *hwrte*) are (jointly) irrelevant in the prediction of semester GPA (*termgpa*) in Model 1.

(d) Using any three selection criteria discussed in class, argue whether Model 1 or Model 2 is preferable?

(e) What is the total sum of squares (SST) from Model 1? From Model 2?

(a) $$\frac{\partial \hat{y}}{\partial atndrte} = \hat{\beta}_3 = 0.0101788$$

$$\frac{\partial \hat{y}}{\partial atndrte} = \hat{\beta}_3 + 2\hat{\beta}_{33}\, atndrte + \hat{\beta}_{34}\, hwrte$$

$$= 0.00486 + 2 \cdot 0.000108\, atndrte + (-0.000128?)\, hwrte$$

(b) $H_0: \beta_{33} = \beta_{44} = \beta_{34} = 0$

H_1: H_0 is not true

model 1 : restricted model

model 2 : unrestricted model

$$F = \frac{(SSR_R - SSR_u)/3}{SSR_u/(674 - 8)} \sim F_{3,666}$$

$$= \frac{(152.226 - 151.4907)/3}{151.4907/666}$$

if $F > F_{(\alpha),3,666}$ $\Rightarrow$ reject H_0

(c) $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$

H_1: H_0 is not true

restricted model

$y = \alpha + u$

unrestricted model

model 1

$$F = \frac{(SSR_R - SSR_u)/4}{SSR_u/(674-5)} \sim F_{4,669}$$

$= 221.436$ (from the table)

w/ p-value < 0.05 ⇒ reject H_0

(d) R^2, SSR, $\hat{\sigma}$, LL ⇒ model 2

AIC, SC, HQ ⇒ model 1

(e) $SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$ does not depend upon the model so the same in both

$$\hat{\sigma}_y = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2} \Rightarrow (n-1)\hat{\sigma}^2 = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

$(673)(0.477^2) = SST$