

# Economics 471: Introductory Econometrics

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Midterm I

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider the model  $y_i = \beta x_i + u_i$  for  $i = 1, 2, \dots, n$ . With this information, answer the following:
  - (a) Write down the sample moment condition for the method of moments estimator.
  - (b) Write down the objective function for the ordinary least-squares estimator.
  - (c) Using the result from (a) *or* (b), derive the estimator of  $\beta$ .
  - (d) Show that the estimator from part (c) is a consistent estimator of  $\beta$ .
  - (e) Suppose the true model is  $y_i = \alpha + \beta x_i + u_i$ . Derive the bias of the estimator from part (c).

2. Consider the model  $y_i = \alpha + \beta x_i + u_i$  for  $i = 1, 2, \dots, n$  and the corresponding residuals from that model  $\hat{u}_i = y_i - \hat{y}_i = y_i - \hat{\alpha} - \hat{\beta}x_i$ . With this information, answer the following:

- (a) Write down the Gauss-Markov Assumptions.
- (b) Using the Gauss-Markov Assumptions you wrote down in part (a), starting with the second equality sign, list which assumption(s) are being employed (if any) to move from one line to the next in the proof below.

$$\begin{aligned} E(\hat{u}_i|x) &= E(y_i - \hat{\alpha} - \hat{\beta}x_i|x) \\ &= E[(\alpha + \beta x_i + u_i) - \hat{\alpha} - \hat{\beta}x_i|x] \\ &= \alpha + \beta E(x_i|x) + E(u_i|x) - E(\hat{\alpha}|x) - E(\hat{\beta}x_i|x) \\ &= \alpha + \beta x_i + E(u_i|x) - E(\hat{\alpha}|x) - E(\hat{\beta}|x)x_i \\ &= \alpha + \beta x_i + E(u_i|x) - \alpha - \beta x_i \\ &= E(u_i|x) \\ &= 0 \end{aligned}$$

- (c) What does the proof in part (b) show?
- (d) Which assumption is not needed?
- (e) Why is the assumption in part (d) unnecessary?

3. Consider the gretl output below on the regression of the log price (lprice) of a house on the log square footage (lsqrft) of the house. With this information, answer the following:

Model 1: OLS, using observations 1–88

Dependent variable: lprice

	Coefficient	Std. Error	<i>t</i> -ratio	p-value
const	−0.975	0.641	−1.521	0.132
lsqrft	0.872	0.084	10.310	0.000
Mean dependent var	5.633	S.D. dependent var	0.303	
Sum squared resid	3.583	S.E. of regression	0.204	
$R^2$	0.552	Adjusted $R^2$	0.547	
$F(1, 86)$	106.389	P-value( $F$ )	0.000	
Log-likelihood	15.971	Akaike criterion	−27.943	
Schwarz criterion	−22.988	Hannan–Quinn	−25.947	

- Interpret the coefficient on lsqrft. Is this result intuitive?
- Test the null hypothesis that the slope is zero, i.e.,  $H_0 : \beta = 0$ .
- What proportion of the total variation in lprice can be explained by the model?
- Test the null hypothesis that the regression can be run through the origin.
- Using the conclusion from part (d), draw the regression (fitted) line on a graph.