Economics 471: Econometrics

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Midterm I – Answers

1. (a) $\widehat{\beta} = \frac{\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}{\sum_{i=1}^{n}(x_i-\overline{x})^2} = \frac{3446.226}{19122.32} = 0.1802201$, $\widehat{\alpha} = \overline{y} - \widehat{\beta}\overline{x} = \frac{1945.26}{274} - 0.1802201\frac{1774.00}{274} = 5.93266$

   (b) The estimate 0.1802201 of $\widehat{\beta}$ means that an increase in firm tenure x_i of 1 year is associated on average with an increase in male employee's hourly wage rate equal to 0.18022 dollars per hour.

   (c) $\widehat{\sigma}^2 = \frac{1}{n-1-1}\sum_{i=1}^{n}\widehat{u}_i^2 = \frac{1}{272}4105.297 = 15.093$

   (d) $Var\left(\widehat{\beta}\right) = \frac{\widehat{\sigma}^2}{\sum_{i=1}^{n}(x_i-\overline{x})^2} = \frac{15.093}{19122.32} = 0.00078929$

   (e) $R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{\sum_{i=1}^{n}\widehat{u}_i^2}{\sum_{i=1}^{n}(y_i-\overline{y})^2} = 1 - \frac{4105.297}{4726.377} = 1 - 0.8686 = 0.1314$. The value of 0.1314 indicates that 13.14 percent of the total sample (or observed) variation in y_i (employees' hourly wage rates) is attributed to, or explained by, the sample regression function of the regressor x_i (firm tenure).

   (f) $t_{\widehat{\beta}} = \frac{\widehat{\beta}-0}{se\left(\widehat{\beta}\right)} = \frac{0.18022-0}{\sqrt{15.093}} = \frac{0.18022}{0.0280943} = 6.4148$. Since $6.4148 > 2$ then we reject the null that the true value of the coefficient is equal to zero.

2. (a) $\sum_{i=1}^{n}\widehat{u}_i^2 = \sum_{i=1}^{n}\left(y_i - \widehat{\beta}x_i\right)^2 \Rightarrow \frac{\partial\sum_{i=1}^{n}\widehat{u}_i^2}{\partial\widehat{\beta}} = -2\sum_{i=1}^{n}\left(y_i - \widehat{\beta}x_i\right)x_i = 0 \Rightarrow \widehat{\beta} = \sum_{i=1}^{n}y_ix_i/\sum_{i=1}^{n}x_i^2$

   (b) $E\left(\widehat{\beta}\right) = E\left(\sum_{i=1}^{n}y_ix_i/\sum_{i=1}^{n}x_i^2\right) = E\left[\sum_{i=1}^{n}(\beta x_i + u_i)x_i/\sum_{i=1}^{n}x_i^2\right]$
   $= E\left[\left(\sum_{i=1}^{n}\beta x_i^2/\sum_{i=1}^{n}x_i^2\right) + \left(\sum_{i=1}^{n}u_ix_i/\sum_{i=1}^{n}x_i^2\right)\right] = \beta + E\left(\sum_{i=1}^{n}u_ix_i/\sum_{i=1}^{n}x_i^2\right)$
   $= \beta + 1/\sum_{i=1}^{n}x_i^2\left[\sum_{i=1}^{n}E(u_ix_i)\right] = \beta$

   (c) $V\left(\widehat{\beta}\right) = V\left(\sum_{i=1}^{n}y_ix_i/\sum_{i=1}^{n}x_i^2\right) = 0+V\left(\sum_{i=1}^{n}u_ix_i/\sum_{i=1}^{n}x_i^2\right) = 1/\left(\sum_{i=1}^{n}x_i^2\right)^2V\left(\sum_{i=1}^{n}u_ix_i\right) = 1/\left(\sum_{i=1}^{n}x_i^2\right)^2\sum_{i=1}^{n}V(u_i)x_i^2 = \sigma^2/\sum_{i=1}^{n}x_i^2$

3. (a) An extra hour spent on homework will result in an additional 60 points on the exam.

   (b) 100/60 = 1 2/3 hours

   (c) $H_0 : \beta = 0$

   (d) $t_\beta = \frac{\widehat{\beta}-0}{se\left(\widehat{\beta}\right)} = \frac{60.3065-0}{0.594503} = 101.4320$

   (e) $R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{2822953}{341552} = -7.26508$ (noting that this value does have much meaning). We can obtain as $SST = \sum_{i=1}^{n}(y_i - \overline{y})^2$. Using the data from $\sigma_y = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \overline{y})^2} \Rightarrow SST = (n-1)\sigma_y^2 = 3732(9.566599) = 341552$