Economics 471: Econometrics

Department of Economics, Finance and Legal Studies

University of Alabama

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Midterm I – Answers

1. (a) i. The model is correctly specified, $y = \alpha + \beta x + u$
   ii. The error is random, $E(u) = 0$
   iii. The error is uncorrelated with the regressor, $E(u|x) = 0$
   iv. The regressor is not a constant, $\sum_{i=1}^{n} (x_i - \overline{x})^2 > 0$
   v. The error is homoskedastic, $V(u|x) = \sigma^2$

   (b) The first four assumptions guarantee that OLS is unbiased

   (c) The first five assumptions guarantee that OLS is BLUE (best linear unbiased estimator)

2. (a) i. $E(u|x) = E(ux) = 0$ or $\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \widehat{\beta}x_i\right)x_i = 0$
   ii. $\sum_{i=1}^{n}\widehat{u}_i^2 = \sum_{i=1}^{n}\left(y_i - \widehat{\beta}x_i\right)^2$
   iii. From (ii), $\frac{\partial \sum_{i=1}^{n}\widehat{u}_i^2}{\partial \widehat{\beta}} = -2\sum_{i=1}^{n}\left(y_i - \widehat{\beta}x_i\right)x_i = 0 \Longrightarrow \sum_{i=1}^{n}\left(y_i - \widehat{\beta}x_i\right)x_i = 0$ (which is identical – once we divide by n – to i and hence we can answer it with both), solving for $\widehat{\beta}$ yields $\widehat{\beta} = \sum_{i=1}^{n} y_i x_i / \sum_{i=1}^{n} x_i^2$

   (b) i. $E(u) = 0$ or $\frac{1}{n}\sum_{i=1}^{n}(y_i - \widehat{\alpha}) = 0$
   ii. $\sum_{i=1}^{n}\widehat{u}_i^2 = \sum_{i=1}^{n}(y_i - \widehat{\alpha})^2$
   iii. From (ii), $\frac{\partial \sum_{i=1}^{n}\widehat{u}_i^2}{\partial \widehat{\alpha}} = -2\sum_{i=1}^{n}(y_i - \widehat{\alpha}) = 0 \Longrightarrow \sum_{i=1}^{n}(y_i - \widehat{\alpha}) = 0$ (which is identical – once we divide by n – to i and hence we can answer it with both), solving for $\widehat{\alpha}$ yields $\widehat{\alpha} = \frac{1}{n}\sum_{i=1}^{n} y_i = \overline{y}$

3. (a) Increasing the class size by student increases the test score by 0.2198 percent

   (b) $SSR = \sum_{i=1}^{n}\widehat{u}_i^2 = 132.4870$

(c) $SST = \sum_{i=1}^{n} (y_i - \overline{y})^2$ $R^2 = 1 - \frac{SSR}{SST} \Longrightarrow SST = \frac{SSR}{1-R^2} = \frac{132.4870}{1-0.006772} = 113.39$ or it can be obtained by the S.D. of the dependent variable as $SST = (n-1)\left(\widehat{\sigma}_y^2\right) = 3732\left(0.189056^2\right) = 113.39$

(d) $SSE = \sum_{i=1}^{n} (\widehat{y}_i - \overline{y})^2$ $SST = SSE + SSR \Longrightarrow SSE = SST - SSR = 133.39 - 132.4870 = 0.93$

(e) Each would be multiplied by 100 $(= 10^2)$ – for example, $SST = \sum_{i=1}^{n} (y_i - \overline{y})^2$ and $\sum_{i=1}^{n} (10y_i - 10\overline{y}) = 100 \sum_{i=1}^{n} (y_i - \overline{y})^2 = 100 \times SST$. However, this would result in no change in R^2.

(f) There would be no change in each of these. Again, R^2 will not change.