Economics 471: Introductory Econometrics

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Midterm I – Answers

1. (a) $E\left(\widetilde{\beta}\right) = E\left[\frac{\sum_{i=1}^{n}(y_i-\overline{y})(x_i-\overline{x})}{\sum_{i=1}^{n}(x_i-\overline{x})^2}\right] = E\left[\frac{\sum_{i=1}^{n}y_i(x_i-\overline{x})}{\sum_{i=1}^{n}(x_i-\overline{x})^2}\right] = E\left[\frac{\sum_{i=1}^{n}(\beta x_i+u_i)(x_i-\overline{x})}{\sum_{i=1}^{n}(x_i-\overline{x})^2}\right] = E\left[\frac{\beta\sum_{i=1}^{n}x_i(x_i-\overline{x})+\sum_{i=1}^{n}u_i(x_i-\overline{x})}{\sum_{i=1}^{n}(x_i-\overline{x})^2}\right]$

$= E\left[\frac{\beta\sum_{i=1}^{n}(x_i-\overline{x})(x_i-\overline{x})+\sum_{i=1}^{n}u_i(x_i-\overline{x})}{\sum_{i=1}^{n}(x_i-\overline{x})^2}\right] = \beta + E\left[\frac{\sum_{i=1}^{n}u_i(x_i-\overline{x})}{\sum_{i=1}^{n}(x_i-\overline{x})^2}\right] = \beta + E\left[\frac{\sum_{i=1}^{n}u_i(x_i-\overline{x})}{\sum_{i=1}^{n}(x_i-\overline{x})^2}\right]$

$= \beta + \frac{1}{\sum_{i=1}^{n}(x_i-\overline{x})^2}\sum_{i=1}^{n}E\left[u_i\left(x_i-\overline{x}\right)\right] = \beta$

(b) Given that $E\left(\widetilde{\beta}\right) = \beta$, the bias is zero.

(c) This estimator is the same as that from class and hence the variance is $\frac{\sigma^2}{\sum_{i=1}^{n}(x_i-\overline{x})^2}$.

2. (a) $\ln(y_i) = \ln\alpha + \beta\ln x_i + u_i$

(b) $\ln\alpha$, β, and u_i, respectively

(c) $\widehat{\alpha} = \overline{ln\,(y)} - \widehat{\beta}\overline{ln\,(x)}$

(d) $\widehat{\beta} = \frac{\sum_{i=1}^{n}\left(\ln y_i-\overline{\ln y}\right)\left(\ln x_i-\overline{\ln x}\right)}{\sum_{i=1}^{n}\left(\ln x_i-\overline{\ln x}\right)^2}$

(e) Given that we have logs on each side of the equation, the slope coefficient is interpreted as an elasticity.

3. (a) This is our left-hand-side variable, in this case testscores

(b) We estimate the model via ordinary least-squares

(c) Our sample size is $n = 3733$

(d) Our intercept parameter estimate $\widehat{\alpha} = \overline{y} - \widehat{\beta}\overline{x}$

(e) Our slope parameter estimate $\widehat{\beta} = \frac{\sum_{i=1}^{n}(y_i-\overline{y})(x_i-\overline{x})}{\sum_{i=1}^{n}(x_i-\overline{x})^2}$

(f) $se\left(\widehat{\alpha}\right) = \sqrt{\frac{\frac{\widehat{\sigma}^2}{n}\sum_{i=1}^{n} x_i^2}{\sum_{i=1}^{n}(x_i - \overline{x})^2}}$

(g) $se\left(\widehat{\beta}\right) = \sqrt{\frac{\widehat{\sigma}^2}{\sum_{i=1}^{n}(x_i - \overline{x})^2}}$

(h) $R^2 = \frac{SSE}{SST} = 1 - \frac{SSR}{SST}$

(i) $\widehat{\sigma} = \sqrt{\widehat{\sigma}^2} = \sqrt{\frac{1}{n-2}SSR} = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}\widehat{u}_i^2}$

(j) $SSR = \sum_{i=1}^{n}\widehat{u}_i^2$

(k) $\widehat{\sigma}_y = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2}$