Economics 471: Introductory Econometrics

Department of Economics, Finance and Legal Studies

University of Alabama

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Midterm I - Key

The exam consists of three questions on three pages. Each question is of equal value.

1. Suppose we observe a random sample of data $\{x_i, y_i\}_{i=1}^n$. Consider the model $y_i = \alpha + u_i$, where $u_i \sim (0, \sigma^2)$. With this information, answer the following:

   (a) Derive the ordinary least-squares estimator of α.

   (b) For the estimator derived in part (a), derive its variance.

   (c) What happens to the variance of the estimator in part (b) as the sample size (n) increases?

   (d) What happens to the variance of the estimator in part (b) as the error variance (σ^2) increases?

   (e) What happens to the variance of the estimator in part (b) when the total variation in x ($\sum_{i=1}^n (x_i - \bar{x})^2$) increases?

(a) $\sum_{i=1}^n \hat{u}_i^2 = \sum_{i=1}^n (y_i - \hat{\alpha})^2$

$$\frac{\partial}{\partial \hat{\alpha}} = \frac{-2 \sum_{i=1}^n (y_i - \hat{\alpha})}{-2} = \frac{0}{-2}$$

$$\sum_{i=1}^n (y_i - \hat{\alpha}) = 0$$

$$\sum_{i=1}^n y_i = \sum_{i=1}^n \hat{\alpha}$$

$$\hat{\alpha} = \frac{1}{n} \sum_{i=1}^n y_i$$

$$= \bar{y}$$

(b) $V(\hat{\alpha}) = V\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right)$

$$\overset{iid}{=} \frac{1}{n^2}\sum_{i=1}^{n} V(y_i)$$

$$\overset{(5)}{=} \frac{1}{n^2}\sum_{i=1}^{n} \sigma^2$$

$$= \frac{n}{n^2}\sigma^2$$

$$= \frac{\sigma^2}{n}$$

$$V(y_i) = V(\alpha + u_i) = V(u_i) = \sigma^2$$

(c) as $n \uparrow \Rightarrow V(\hat{\alpha}) \downarrow$

(d) as $\sigma^2 \uparrow \Rightarrow V(\hat{\alpha}) \uparrow$

(e) as $\sum_{i=1}^{n}(x_i - \bar{x})^2 \Rightarrow$ no change in $V(\hat{\alpha})$

2. Consider the population regression function $Y = \alpha + \beta X + U$, where $U \sim (0, \sigma^2)$. Assuming $\alpha > 0$ and $\beta < 0$, in the figure below, perform the following:

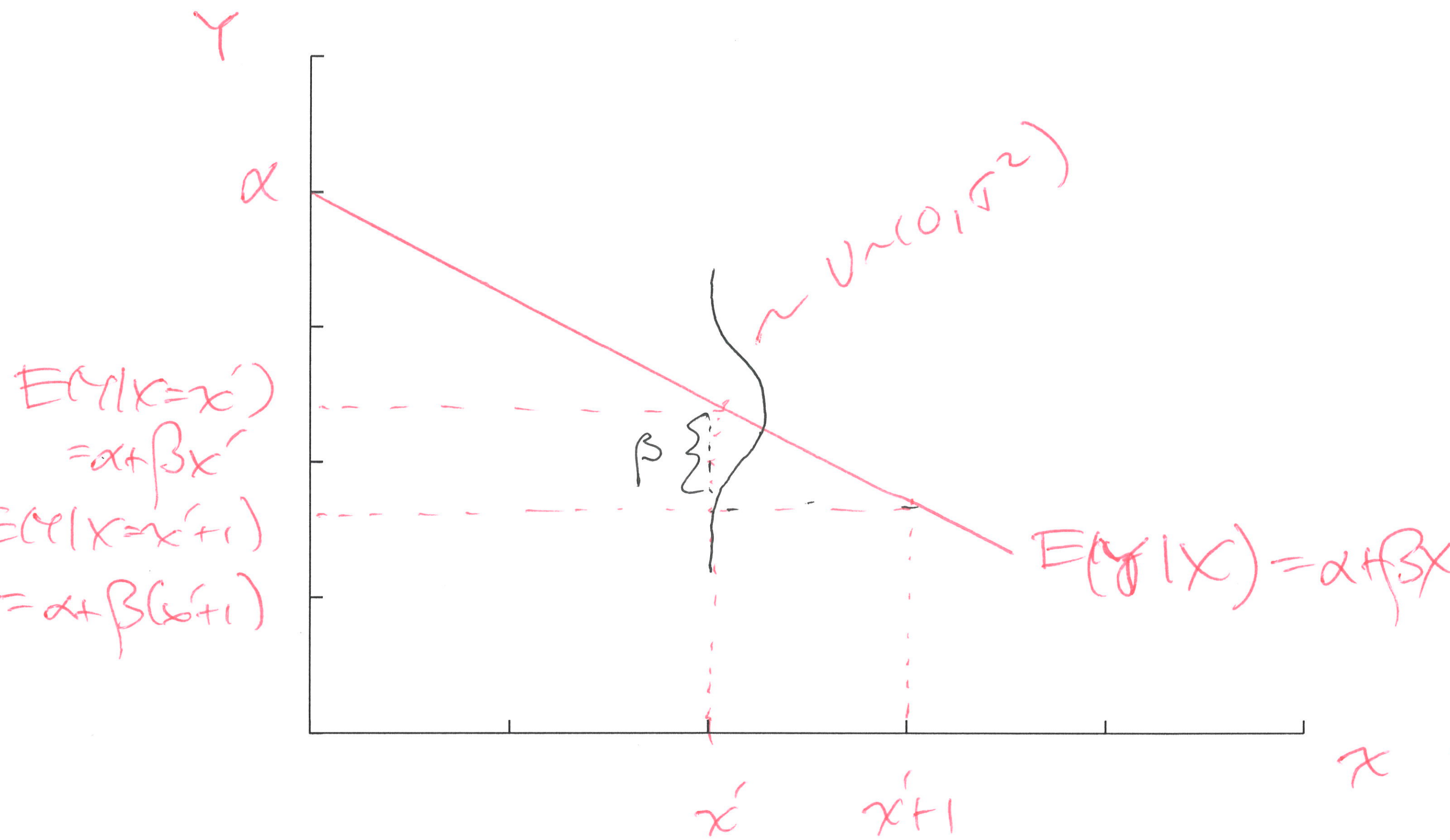
   (a) Label the axes

   (b) Plot and label the population regression line.

   (c) Pick an arbitrary value for $X = x'$ and plot its conditional expecatation (i.e., $E(Y|X = x')$).

   (d) Plot and label the distribution of the error (U) about its conditional expectation for the point you listed in part (c).

   (e) Suppose there was a one unit increase in the value you picked for X in part (b). What is the marginal impact on $E(Y|X = x')$ from that one unit increase? Show this on the figure.



$$\frac{\partial E(Y|X=x')}{\partial x'} = \beta$$

3. Consider the gretl output below for cross-country data on the number of deaths per 100,000 individuals (*deaths*) versus the liters of alcohol consumed from wine, per capita (*alcohol*). With this information, answer the following (give all formulae used):

Model 1: OLS, using observations 1–21

Dependent variable: *deaths*

	Coefficient	Std. Error	t-ratio	p-value
const	876.205	30.4682	28.76	0.0000
alcohol	−16.2635	8.19892	−1.984	0.0619

Mean dependent var	830.0476	S.D. dependent var	96.51864
Sum squared resid	154352.0	S.E. of regression	90.13207
R^2	0.171562	Adjusted R^2	0.127960
$F(1, 19)$	3.934731	P-value(F)	0.061939
Log-likelihood	−123.2736	Akaike criterion	250.5473
Schwarz criterion	252.6363	Hannan–Quinn	251.0006

(a) What is the estimated variance of the number of deaths ($\widehat{\sigma}^2_{deaths}$)?

(b) What is the estimated error variance ($\widehat{\sigma}^2$)?

(c) Interpret the intercept coefficient. Does this appear to be reasonable?

(d) Interpret the slope coefficient. Does this appear to be reasonable?

(e) What percentage of the variation in deaths can be explained by the variation in alcohol consumption?

(a) $\hat{\sigma}_y^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2$

$= (96.51864)^2$

(b) $\hat{\sigma}^2 = \frac{1}{n-2} \sum_{i=1}^{n} \hat{u}_i^2$

$= \frac{1}{19} (154352.0)$

or

$= (90.13207)^2$

(c) if 0 liters of alcohol are consumed, we would expect 876.205 death ~~per capita~~ 100,000 individuals (seems reasonable)

(d) a 1L ↑ in alcohol per capita would decrease the expected number of deaths by 16.2635 people per 100,000 individuals (more alcohol less death seems unreasonable, but more wine may be correlated w/ more income)

(e) $R^2 = 0.171562$

or 17.1562% of the variation