Economics 471: Introductory Econometrics

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Midterm I -Key

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider the model $y_i = \alpha + u_i$, $i = 1, 2, \ldots, n$.

   (a) Using method of moments, derive the estimator of α.

   (b) Using least-squares, derive the estimator of α.

   (c) Show that the estimator you obtained in (a) or (b) is an unbiased estimator of α.

   (d) Suppose the true data generating process is $y_i = \alpha + \beta x_i + u_i$. What is the expected value of your estimator from part (a) or (b) conditional upon x?

   (e) Considering your result from part (d), under what condition(s) will that estimator be unbiased?

(a) $E(u)=0 \Rightarrow E(y-\alpha)=0$, $\frac{1}{n}\sum_{i=1}^{n}(y_i-\hat{\alpha})=0 \Rightarrow$

$\frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{n}\sum_{i=1}^{n}\hat{\alpha} \Rightarrow \hat{\alpha} = \frac{1}{n}\sum_{i=1}^{n} y_i = \bar{y}$

(b) $\sum_{i=1}^{n}\hat{u}_t^2 = \sum_{i=1}^{n}(y_i-\hat{\alpha})^2 \Rightarrow \frac{\partial \sum_{i=1}^{n}\hat{u}_i^2}{\partial \hat{\alpha}} = -2\sum_{i=1}^{n}(y_i-\hat{\alpha})=0$

$\Rightarrow \sum_{i=1}^{n}(y_i-\hat{\alpha})=0 \Rightarrow \sum_{i=1}^{n} y_i = \sum_{i=1}^{n}\hat{\alpha} \Rightarrow \sum_{i=1}^{n} y_i = n\hat{\alpha}$

$\Rightarrow \hat{\alpha} = \frac{1}{n}\sum_{i=1}^{n} y_i = \bar{y}$

(c) $E(\hat{\alpha}|x) = E\left(\frac{1}{n}\sum_{i=1}^{n} y_i|x\right) = E\left(\frac{1}{n}\sum_{i=1}^{n}\alpha + u_i|x\right) = \alpha + \sum_{i=1}^{n}E(u_i|x)$

$= \alpha$

(d) $E(\hat{\alpha}|x) = E\left[\frac{1}{n}\sum_i y_i | x\right] = E\left[\frac{1}{n}\sum_{i=1}^{n}(\alpha + \beta x_i + u_i) | x\right]$

$= \alpha + \beta \bar{x} + \frac{1}{n}\sum_{i=1}^{n} E(u_i|x_i) = \alpha + \beta\bar{x}$

(e) biased unless $\beta = 0$ or $\bar{x} = 0$

2. Consider the model $y_i = \alpha + \beta x_i + u_i$ for $i = 1, 2, \ldots, n$ and the least-squares parameter estimate of $\hat{\beta}$. Utilizing the proof listed below, answer the following:

$$
\begin{aligned}
E\left(\hat{\beta}|x\right) &\overset{(4)}{=} E\left(\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}|x\right) \\
&\overset{\times}{=} E\left(\frac{\sum_{i=1}^{n}(x_i-\bar{x})y_i}{\sum_{i=1}^{n}(x_i-\bar{x})^2}|x\right) \\
&\overset{(1)}{=} E\left(\frac{\sum_{i=1}^{n}(x_i-\bar{x})(\alpha+\beta x_i+u_i)}{\sum_{i=1}^{n}(x_i-\bar{x})^2}|x\right) \\
&\overset{\times}{=} E\left(\frac{\alpha\sum_{i=1}^{n}(x_i-\bar{x})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}+\frac{\beta\sum_{i=1}^{n}(x_i-\bar{x})(x_i-\bar{x})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}+\frac{\sum_{i=1}^{n}u_i(x_i-\bar{x})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}|x\right) \\
&\overset{\times}{=} \beta + E\left(\frac{\sum_{i=1}^{n}u_i(x_i-\bar{x})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}|x\right) \\
&\overset{(2,3)}{=} \beta
\end{aligned}
$$

(a) Write down the Gauss-Markov Assumptions.

(b) Using the Gauss-Markov Assumptions you wrote down in part (a), list which assumption(s) are being employed (if any) to move from one line to the next in the proof below.

(c) What does the proof above prove?

(d) Formally show how to move from the fourth to fifth equality sign (i.e., show that $\alpha\sum_{i=1}^{n}(x_i-\bar{x}) = 0$).

(e) Formally show how to move from the first to second equality sign (i.e., show that $\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y}) = \sum_{i=1}^{n}(x_i-\bar{x})y_i$).

(a) (1) $y_i = \alpha + \beta x_i + u_i$

(2) $E(u) = 0$

(3) $E(u|x) = 0$

(4) $\sum_{i=1}^{n}(x_i - \bar{x})^2 > 0$

(5) $V(u|x) = \sigma^2$

(b) see above

(c) $\hat{\beta}$ is an unbiased estimate of β

(d) $\alpha \sum_{i=1}^{n} (x_i - \bar{x}) = 0$

sufficient to show $\sum_{i=1}^{n} (x_i - \bar{x}) = 0$

note that $\frac{1}{n} \sum_{i=1}^{n} x_i = \bar{x}$ by definition

$$\sum_{i=1}^{n} (x_i - \bar{x}) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \bar{x}$$

$$= n\bar{x} - n\bar{x}$$

$$= 0$$

(e) $$\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x}) = \sum_{i=1}^{n} y_i (x_i - \bar{x}) - \sum_{i=1}^{n} \bar{y}(x_i - \bar{x})$$

$$= \sum_{i=1}^{n} y_i (x_i - \bar{x}) - \bar{y} \sum_{i=1}^{n} (x_i - \bar{x})$$

$$= \sum_{i=1}^{n} y_i (x_i - \bar{x})$$

because $\sum_{i=1}^{n} (x_i - \bar{x}) = 0$

as shown in Part (d)

3. Consider the gretl output below for the regression of college GPA (colGPA) on high school GPA (hsGPA). With this information, answer the following:

Model 1: OLS, using observations 1–141
Dependent variable: colGPA

	Coefficient	Std. Error	t-ratio	p-value
const	1.415	0.307	4.611	0.0000
hsGPA	0.482	0.090	5.371	0.0000

Mean dependent var	3.056	S.D. dependent var	0.372
Sum squared resid	16.071	S.E. of regression	0.340
R^2	0.171	Adjusted R^2	0.165
$F(1, 139)$	28.845	P-value(F)	3.21e–07
Log-likelihood	−46.962	Akaike criterion	97.925
Schwarz criterion	103.822	Hannan–Quinn	100.321

(a) Interpret the intercept coefficient. Is this result intuitive?

(b) Interpret the slope coefficient. Is this result intuitive?

(c) What proportion of the variation in college GPA can be explained by the model?

(d) What high school GPA will predict a perfect college GPA (i.e., 4.0 college GPA)?

(e) Test the null that high school GPA is irrelevant in predicting college GPA.

(a) if hsGPA = 0 we <u>expect</u> col GPA to be 1.415

(b) if hs GPA ↑ by 1 unit, we <u>expect</u> colGPA to rise by 0.482

(c) $R^2 = 0.171$ or 17.1 %

(d) $\hat{y} = \hat{\alpha} + \hat{\beta} x$

$4.0 = 1.415 + 0.482x \Rightarrow x^* = \frac{4.0 - 1.415}{0.482}$

$= 5.363$

(e) $H_0: \beta = 0$

$H_a: \beta \neq 0$

$$t_\beta = \frac{0.482 - 0}{0.090}$$

$= 5.371$

> 2

$\Rightarrow$ reject H_0

(or note p-value < 0.05)