Economics 471: Introductory Econometrics

Department of Economics, Finance and Legal Studies

University of Alabama

Fall 2021

Midterm I - Key

The exam consists of three questions on three pages. Each question is of equal value.

1. We claimed the explained sum of squares (SSE) plus the residual sum of squares (SRR) was equal to the total sum of squares (SST), i.e., $SSE + SSR = SST$. Noting this claim, answer the following:

   (a) Explicity write down the formula for SST. Briefly state what it represents.

   (b) Explicity write down the formula for SSE. Briefly state what it represents.

   (c) Explicity write down the formula for SSR. Briefly state what it represents.

   (d) We stated that our goodness-of-fit measure can be written as $R^2 = 1 - \frac{SSR}{SST}$ or equivalently as $R^2 = \frac{SSE}{SST}$. Briefly explain what each represents.

   (e) Prove the two versions in part (d) are equivalent, i.e., $1 - \frac{SSR}{SST} = \frac{SSE}{SST}$.

(a) $SST = \sum_{i=1}^{n}(y_i - \bar{y})^2$ total variation in y (at one)

(b) $SSE = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2$ total variation in $\hat{y}$ (explained)

(c) $SSR = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{n}\hat{u}_i^2$ total variation in $\hat{u}$ (unexplained)

(d) $R^2 = 1 - \frac{SSR}{SST}$ 1 - ratio of unexplained to total variation

$R^2 = \frac{SSE}{SST}$ ratio of explained to total variation

both $0 \leq R^2 \leq 1$

(e) $1 - \frac{SSR}{SST} = \frac{SST}{SST} - \frac{SSR}{SST} = \frac{SST - SSR}{SST} = \frac{SSE}{SST}$

2. Consider the population regression function $y = \alpha + u$, where $u \sim (0, \sigma^2)$. Assuming $\alpha > 0$, in the figure below, perform the following:

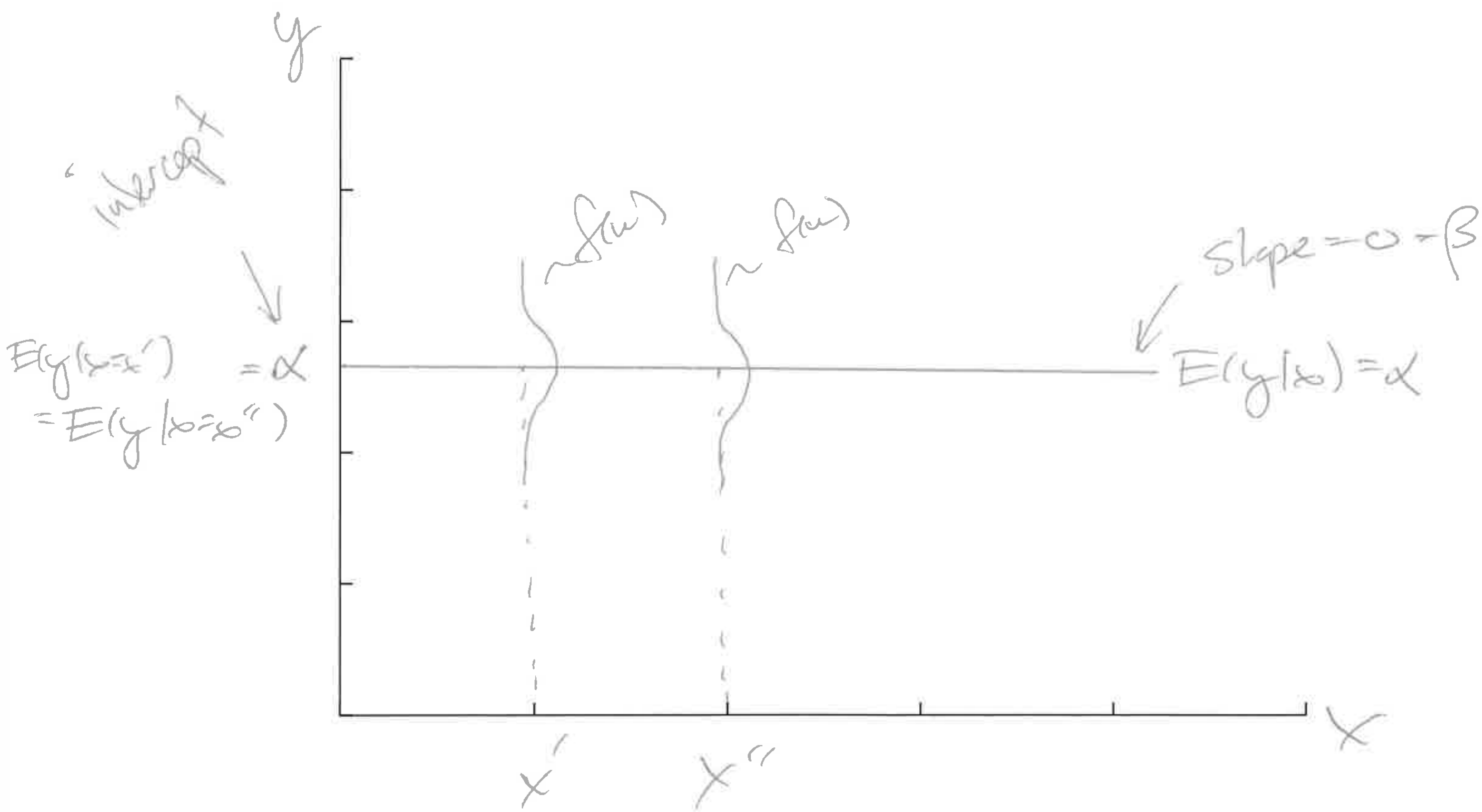
    (a) Label the axes.

    (b) Plot the population regression function.

    (c) Label the intercept and slope of the function drawn in part (b).

    (d) Pick two specific values for x, plot their conditional expectations (i.e., $E(y|x)$).

    (e) Plot the distribution of the error (u) for each of the points you listed in part (d).



3. Consider the gretl output below on the regression of the number of cigarettes (cigs) smoked per day on log income (lincome). With this information, answer the following:

Model 1: OLS, using observations 1–807

Dependent variable: cigs

	Coefficient	Std. Error	t-ratio	p-value
const	−3.62159	6.57698	−0.5506	0.5820
lincome	1.27054	0.677100	1.876	0.0610

Mean dependent var	8.686493	S.D. dependent var	13.72152
Sum squared resid	151092.8	S.E. of regression	13.70011
R^2	0.004355	Adjusted R^2	0.003118
$F(1, 805)$	3.521023	P-value(F)	0.060957
Log-likelihood	−3256.327	Akaike criterion	6516.654
Schwarz criterion	6526.040	Hannan–Quinn	6520.258

(a) Interpret the coefficient on lincome. Is this result intuitive?

(b) Interpret the intercept coefficient. Is this result intuitive?

(c) Test the null hypothesis that the intercept is zero, i.e., $H_0 : \alpha = 0$.

(d) Suppose we multipled cigs by 10 (for all observations). What will happen to the estimated intercept and slope parameters (be explicit)?

(e) Suppose we multiplied cigs by 10 (for all observations). What will happen to SST? What will happen to R^2?

(a) 1% ↑ in income ⇒ 1.27 more cigs smoked per day
ok if cigs are a normal good

(b) if earn 0, you smoke −3.62 cigs per day
not sure what that means (illogical)

(c) $H_0: \alpha = 0$, $H_1: \alpha \neq 0$

$t = \frac{\hat{\alpha} - 0}{se(\hat{\alpha})} = \frac{-3.62159 - 0}{6.57698} = |-0.5506| < 2 \Rightarrow$ fail to reject H_0, or not pvalue $= 0.5820 > 0.05$

(d) If y multiplied by c, $\hat{\alpha}$ & $\hat{\beta}$ are multiplied by c

$$\tilde{\alpha} = 10 \cdot (-3.62459) = -36.2459$$

$$\tilde{\beta} = 10 \cdot (1.27054) = 12.7054$$

(c) $SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$

$cy_i \Rightarrow$

$$\sum_{i=1}^{n} (cy_i - c\bar{y})^2 = c^2 \sum_{i=1}^{n} (y_i - \bar{y})^2 = c^2 SST$$

$$SSE = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2$$

$cy_i \Rightarrow$

$$\sum_{i=1}^{n} (c\hat{y}_i - c\bar{y})^2 = c^2 \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2 = c^2 SSE$$

$$R^2 = \frac{SSE}{SST}$$

$cy_i \Rightarrow$

$$\frac{c^2 SSE}{c^2 SST} = \frac{SSE}{SST} = R^2$$