Economics 471: Introductory Econometrics

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Midterm I – Answer Key

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider a regression model through the origin: $y_i = \beta x_i + u_i,\ i = 1, 2, \ldots, n$, where $y_i > 0$ and $x_i > 0$ for all i, and two competing estimators of the slope parameter

$$\widehat{\beta} = \frac{\sum_{i=1}^n y_i x_i}{\sum_{i=1}^n x_i^2}$$

$$\widetilde{\beta} = \frac{\sum_{i=1}^n y_i}{\sum_{i=1}^n x_i}$$

Given this information, answer the following:

(a) Derive the expected value of $\widehat{\beta}$.

(b) Derive the variance of $\widehat{\beta}$.

(c) Derive the expected value of $\widetilde{\beta}$.

(d) Derive the variance of $\widetilde{\beta}$.

(e) Is $\widehat{\beta}$ a consistent estimator of β? Is $\widetilde{\beta}$ a consistent estimator of β? Can anything be said about the relative efficiency of these estimators?

(a) $E(\hat\beta) = E\left(\frac{\sum y_i x_i}{\sum x_i^2}\right) = E\left(\frac{\sum(\beta x_i + u_i)x_i}{\sum x_i^2}\right) = \beta + E\left(\frac{\sum u_i x_i}{\sum x_i^2}\right) = \beta$

(b) $V(\hat\beta) = V\left(\frac{\sum y_i x_i}{\sum x_i^2}\right) = V\left(\frac{\sum(\beta x_i + u_i)x_i}{\sum x_i^2}\right) = V\left(\beta + \frac{\sum u_i x_i}{\sum x_i^2}\right)$

$\overset{iid}{=} 0 + \frac{1}{(\sum x_i^2)^2} \sum x_i^2 V(u_i) = \sigma^2 / \sum x_i^2$

(c) $E(\tilde\beta) = E\left(\frac{\sum y_i}{\sum x_i}\right) = E\left(\frac{\sum \beta x_i + u_i}{\sum x_i}\right) = \beta + E\left(\frac{\sum u_i}{\sum x_i}\right) = \beta$

(d) $V(\tilde\beta) = V\left(\frac{\sum y_i}{\sum x_i}\right) = V\left(\frac{\sum \beta x_i + u_i}{\sum x_i}\right) = V\left(\beta + \frac{\sum u_i}{\sum x_i}\right)$

$\overset{iid}{=} 0 + \frac{1}{(\sum x_i)^2} \sum V(u_i) = \frac{n\sigma^2}{(\sum x_i)^2}$

(e) yes, yes (given $x_i > 0\ \forall i$)

$V(\hat\beta) = \frac{\sigma^2}{\sum x_i^2} \gtreqless \frac{n\sigma^2}{(\sum x_i)^2} = V(\tilde\beta)$?

2. Consider the population regression function $y = \alpha + \beta x + u$. Assuming $\alpha > 0$ and $\beta < 0$, in a figure below, perform the following:

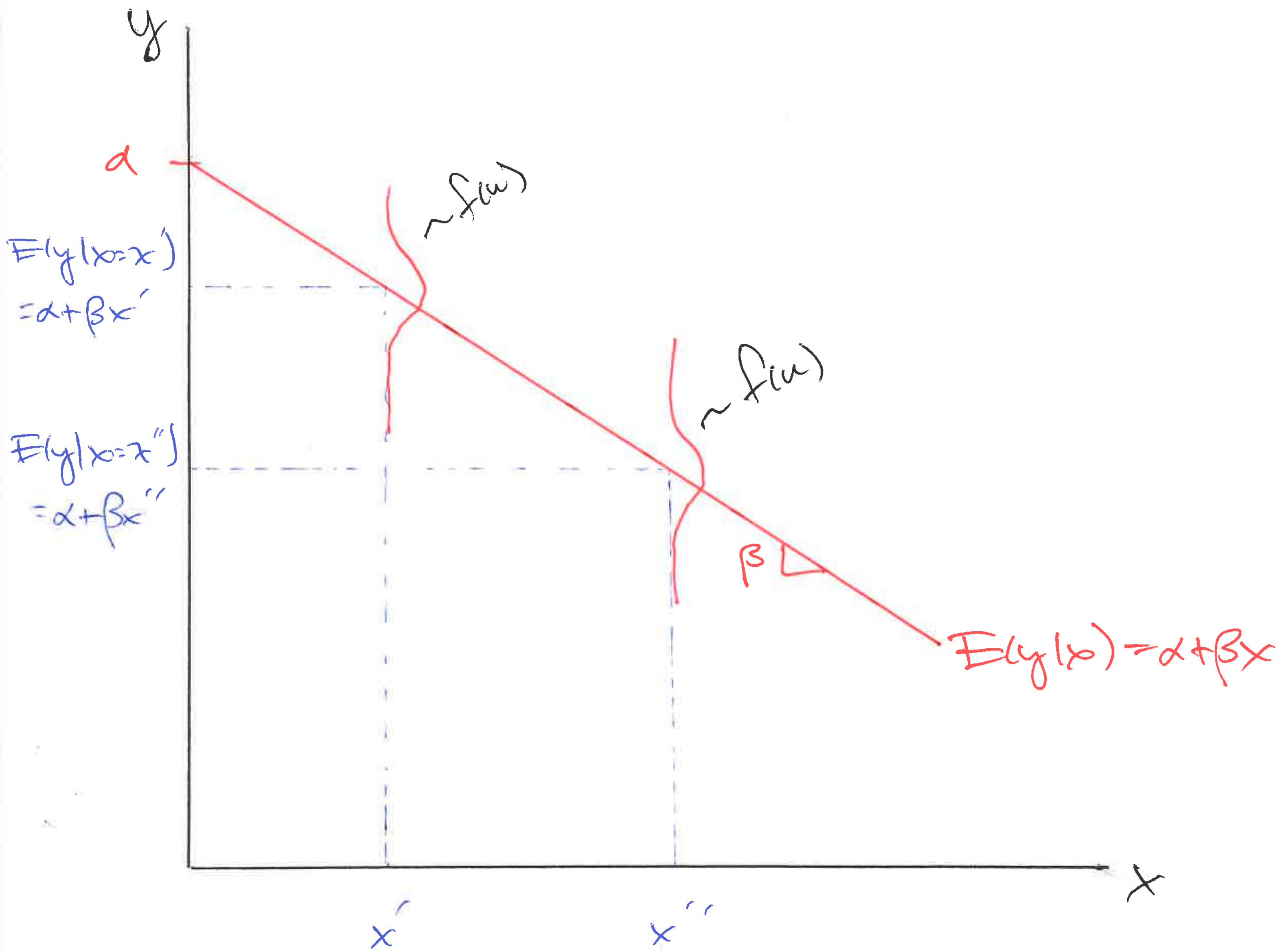
   (a) Label the axes.

   (b) Plot the population regression line.

   (c) Label the intercept, slope and line in part (b).

   (d) Pick two values for x, plot their conditional expectations (i.e., $E(y|x)$).

   (e) Assuming homoskedastic errors, plot the distribution of the error (u) for each of the points you listed in part (d).



3. Consider the relationship between wages and education. Sample output from gretl from the regression of log wages ($lwage$) on education ($educ$): $lwage = \alpha + \beta educ + u$ is listed below

Model 1: OLS, using observations 1–526
Dependent variable: lwage

	Coefficient	Std. Error	t-ratio	p-value
const	0.583773	0.0973358	5.998	0.0000
educ	0.082744	0.0075667	10.940	0.0000

Mean dependent var	1.623268	S.D. dependent var	0.531538
Sum squared resid	120.7691	S.E. of regression	0.480079
R^2	0.185806	Adjusted R^2	0.184253
$F(1, 524)$	119.5816	P-value(F)	3.27e–25
Log-likelihood	−359.3781	Akaike criterion	722.7562
Schwarz criterion	731.2868	Hannan–Quinn	726.0963

Using this information, answer the following:

(a) What is the sample size (n)?

(b) Interpret the coefficient on the slope term.

(c) What percent of the variation in log wages ($lwage$) is explained by the model?

(d) Test the null hypothesis that education is irrelevant.

(e) Test the null hypothesis that we can run the regression through the origin.

(a) $n = 526$

(b) $\hat{\beta} = \frac{\partial \widehat{\ln wage}}{\partial educ} \approx \frac{\% \Delta \widehat{wage}}{\Delta educ}$

↑ in one year of educ ⇒ ↑ in predicted wage by 8.27%

(c) $R^2 = 0.1858$

(d) $H_0: \beta = 0$ vs. $H_1: \beta \neq 0$ $\quad t = \frac{\hat{\beta} - 0}{se(\hat{\beta})} = 10.940 > 2$

⇒ reject H_0 (p-value = 0.0000)

(e) $H_0: \alpha = 0$ vs. $H_1: \alpha \neq 0$ $\quad t = \frac{\hat{\alpha} - 0}{se(\hat{\alpha})} = 5.998 > 2$

⇒ reject H_0 (p-value = 0.0000)