

# Economics 471: Introductory Econometrics

Department of Economics, Finance and Legal Studies

University of Alabama

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## Fundamentals Exam

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider the random variable  $X \sim \left(0, \frac{v}{v-2}\right)$  with probability density function

$$f_X(x) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\Gamma\left(\frac{v}{2}\right)} \left(1 + \frac{x^2}{v}\right)^{-\frac{v+1}{2}}$$

where  $v > 2$ ,  $\Gamma(\cdot)$  is the gamma function and  $-\infty < x < \infty$ .

- (a) What is the expected value of this random variable (state, but do not derive)?
- (b) What is the variance of this random variable (state, but do not derive)?
- (c) If  $f_X(x)$  were integrated from negative infinity to infinity, what would be the resulting outcome?
- (d) Suppose we standardized this random variable ( $X$ ): What is the expected value of this standardized random variable? What is the variance of this standardized random variable?

2. Consider the random variable  $X \sim (\mu_X, \sigma_X^2)$  and the competing estimators for the underlying population average

$$\begin{aligned}\overleftarrow{x} &= \frac{2}{n} \sum_{i=1}^n x_i \\ \tilde{x} &= \frac{1}{2n} \sum_{i=1}^n x_i\end{aligned}$$

for a random sample of independent  $i = 1, 2, \dots, n$  observations.

- (a) What is the expected value of each estimator?
- (b) What is the bias of each estimator?
- (c) What is the variance of each estimator?
- (d) Is either estimator consistent? How do you know?

3. A researcher is using data for a sample of 312 employees to investigate the relationship between hourly sales  $y_i$  (measured in dollars per hour) and education  $x_i$  (measured in years). Preliminary analysis of the sample data produces the following sample information :

$$\begin{aligned}\sum_{i=1}^n y_i &= 1982.14 & \sum_{i=1}^n x_i &= 1776.21 \\ \sum_{i=1}^n y_i^2 &= 19999.86 & \sum_{i=1}^n x_i^2 &= 31114.21 \\ \sum_{i=1}^n (y_i - \bar{y})^2 &= 4874.21 & \sum_{i=1}^n (x_i - \bar{x})^2 &= 18521.25 \\ \sum_{i=1}^n x_i y_i &= 17214.35 & \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) &= 3517.55\end{aligned}$$

Use the above sample information to answer all the following questions. Show explicitly all formulas, but it is not necessary to simplify fractions.

- What is the sample average of hourly sales?
- What is the sample variance of education?
- What is the covariance between hourly sales and education?
- What is the correlation between hourly sales and education?