

Economics 471: Introductory Econometrics

Department of Economics, Finance and Legal Studies

University of Alabama

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Fundamentals Exam

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider the table below which gives the value of the sum ($X = Z_1 + Z_2$) of two fair-sided dice (Z_1, Z_2) thrown simultaneously (hint: Z_1 and Z_2 each have equal probability of each event $1, 2, \dots, 6$ and are independent of one another). With this information, answer the following (*all formulae must be given*):

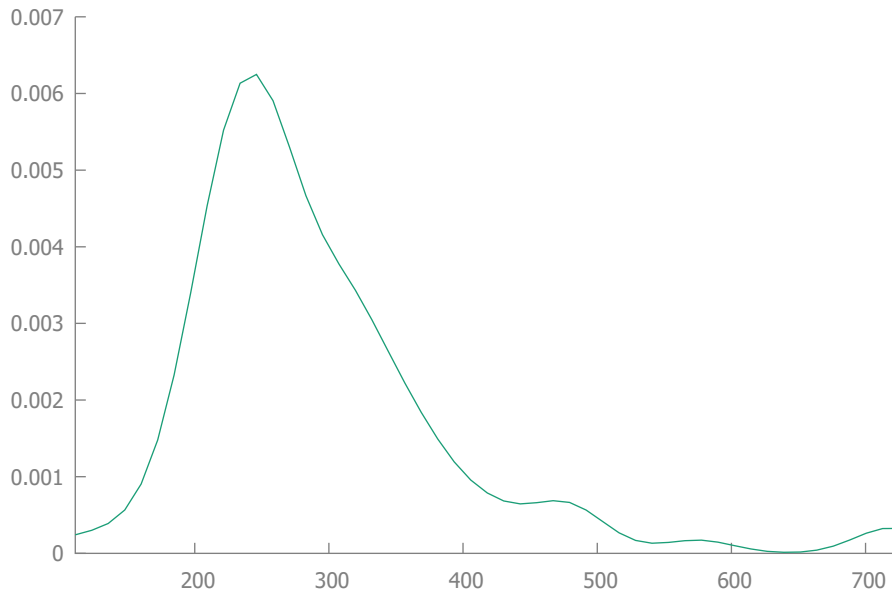
		Z_1					
		1	2	3	4	5	6
Z_2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

- (a) What is the expected value of the first dice ($E(Z_1)$)?
- (b) What is the probability that the sum is 6 ($P(X = 6)$)?
- (c) Draw the probability density function of the sum ($f_X(x)$).
- (d) What is the probability the sum is 6 given the value of the first dice is a 2 ($P(X = 6|Z_1 = 2)$)?
- (e) What is the expected value of the sum given the value of the first dice is a 2 ($E(X|Z_1 = 2)$)?

2. Consider a random variable $X \sim \left(\frac{d_2}{d_2-2}, \frac{2d_2^2(d_1+d_2-2)}{d_1(d_2-2)^2(d_2-4)} \right)$ that comes from the F -distribution with d_1 and d_2 degrees of freedom (for the numerator and denominator, respectively). With this information, answer the following (show your work):

- (a) Draw a reasonable probability density function for X ($f_X(x)$).
- (b) What is the expected value of X ($E(X)$)? What is the variance of X ($V(X)$)?
- (c) Suppose you draw a random sample of data $\{x_i\}_{i=1}^n$ from this distribution. What estimator would you use to estimate the mean (give the formula)?
- (d) What is the expected value you estimator from part (c)?
- (e) What is the variance of your estimator from part (c)?

3. Consider the estimated probability density function below (price of a home – measured in \$1,000s). Using this figure, answer the following:



- (a) Draw a reasonable value for the mean.
- (b) Draw a reasonable value for the median
- (c) Draw a reasonable value for the mode.
- (d) What is the probability that the price of a home is \$250k?
- (e) Draw a reasonable cumulative distribution function that corresponds to this estimated probability density function.