Economics 471: Introductory Econometrics

Department of Economics, Finance and Legal Studies

University of Alabama

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Fundamentals Exam —Key

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider the summation operator

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n$$

the sample mean estimator

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

and a constant c. With this information, show the following results:

(a) $\sum_{i=1}^{n} c = nc$

(b) $\sum_{i=1}^{n} cx_i = c\sum_{i=1}^{n} x_i$

(c) $\sum_{i=1}^{n} c(x_i - \bar{x}) = 0$

(d) $\sum_{i=1}^{n}(y_i - \bar{y})(x_i - \bar{x}) = \sum_{i=1}^{n}(y_i - \bar{y})x_i$

(e) $\sum_{i=1}^{n}(y_i - \bar{y})(x_i - \bar{x}) = (\sum_{i=1}^{n} y_i x_i) - n\bar{y}\bar{x}$

(a) $\sum_{i=1}^{n} c = c + c + c + \cdots + c = nc$

(b) $\sum_{i=1}^{n} cx_i = cx_1 + cx_2 + \cdots + cx_n = c(x_1 + x_2 + \cdots + x_n) = c\sum_{i=1}^{n} x_i$

(c) $\sum_{i=1}^{n} c(x_i - \bar{x}) \overset{\text{via (b)}}{=} c\sum_{i=1}^{n}(x_i - \bar{x}) = c\left[\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \bar{x}\right]$

$= c\left[\sum_{i=1}^{n} x_i - n\bar{x}\right] \overset{n\bar{x} = \sum x_i}{=} c\left[n\bar{x} - n\bar{x}\right] = c \cdot 0 = 0$

(d) $\sum_{i=1}^{n}(y_i - \bar{y})(x_i - \bar{x}) = \sum_{i=1}^{n}(y_i - \bar{y})x_i - \sum_{i=1}^{n}(y_i - \bar{y})\bar{x}$

$= \sum_{i=1}^{n}(y_i - \bar{y})x_i - \bar{x}\sum_{i=1}^{n}(y_i - \bar{y}) \overset{\text{via (c)}}{=} \sum_{i=1}^{n}(y_i - \bar{y})x_i$

(e) $\sum_{i=1}^{n}(y_i - \bar{y})(x_i - \bar{x}) \overset{\text{via (d)}}{=} \sum_{i=1}^{n}(y_i - \bar{y})x_i = \sum_{i=1}^{n} y_i x_i - \sum_{i=1}^{n} \bar{y} x_i$

$= \sum_{i=1}^{n} y_i x_i - \bar{y}\sum_{i=1}^{n} x_i \overset{n\bar{x} = \sum x_i}{=} \sum_{i=1}^{n} y_i x_i - \bar{y}n\bar{x}$

2. Consider a random variable X from a chi-square distribution with J degrees of freedom ($X \sim \chi^2_J$), where we known that a chi-square distribution is defined as a random variable formed from the sums of squares of independent standard normal ($Z \sim N(0,1)$) random variables

$$X = \sum_{j=1}^{J} Z_j^2$$

With this information, answer the following:

(a) What is the domain of X?

(b) Draw a reasonable probability density function for this random variable (label the axes).

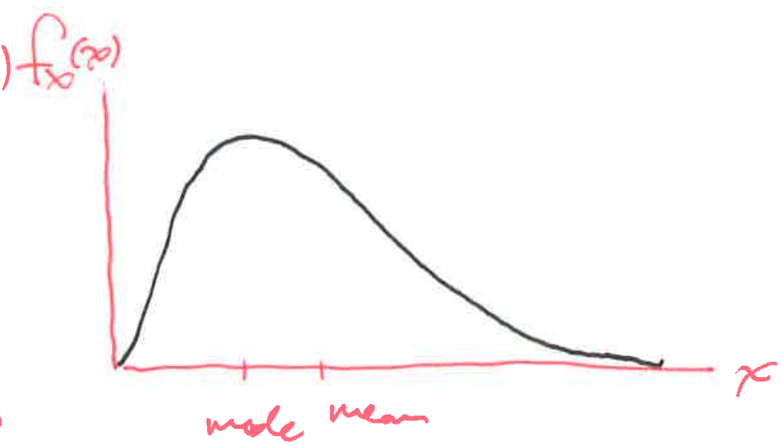
(c) On the figure you drew in part (b), mark a reasonable mean and mode.

(d) Draw a reasonable cummulative distribution function for this random variable (label the axes).

(e) On the figure you drew in part (d), mark a reasonable median.

(a) $0 \leq x < \infty$ (as X is sum of squares)

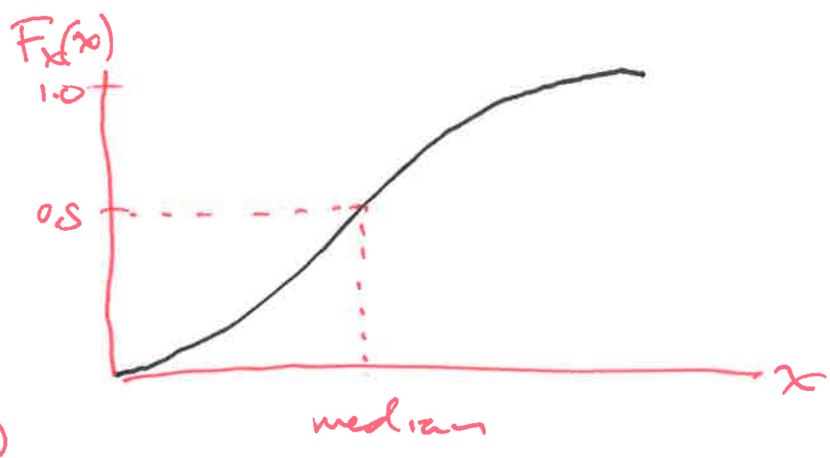
(b)



(must be skewed)

(c)

(d)



(e)

3. The sample output from gretl below includes descriptive statistics and a correlation matrix using data from 1388 mothers. The variables listed are the birthweight of their child in ounces (bwght), local cigarette taxes in cents (cigtax), local cigarette price in cents (cigprice) and number of cigarettes smoked daily during pregnancy (cigs).

Summary Statistics, using the observations 1–1388

Variable	Mean	Median	S.D.	Min	Max
bwght	119.0	120.0	20.4	23.0	271.0
cigtax	19.6	20.0	7.8	2.0	38.0
cigprice	131.0	131.0	10.2	104.0	153.0
cigs	2.1	0.0	6.0	0.0	50.0

Correlation coefficients, using the observations 1–1388

bwght	cigtax	cigprice	cigs	
1.0000	0.0478	0.0492	−0.1508	bwght
	1.0000	0.8759	0.0294	cigtax
		1.0000	0.0097	cigprice
			1.0000	cigs

Using the information above, answer the following (give numerical value and formula when feasible):

(a) What is the (sample) mean cigarette tax (cigtax) in dollars?

(b) What is the 95% confidence interval for the population mean cigarette tax (it is okay to use the rule-of-thumb mentioned in class)?

(c) Write a test statistic for the null hypothesis that the population mean cigarette tax is zero versus the null that it is not zero.

(d) What is the distribution of that test statistic from part (c) under the null hypothesis?

(e) Draw a potential sampling distribution for the test statistic you suggested from part (d). Be sure the label the axes as well as the mean, median and mode values.

(a) mean \$ 0.196 (note: table is in cents)

(b) $P(\bar{x} - t_{\frac{\alpha}{2},n-1} \, s/\sqrt{n} < \mu < \bar{x} + t_{\frac{\alpha}{2},n-1} \, s/\sqrt{n}) = 1-\alpha$

ROT $t_{\frac{\alpha}{2},n-1} = 2$ for $\alpha = 0.05$

$P(\bar{x} - 2 \, s/\sqrt{n} < \mu < \bar{x} + 2 \, s/\sqrt{n}) \approx 0.95$

$P(19.6 - 2 \, 20/\sqrt{1388} < \mu < 19.6 + 2 \, 20/\sqrt{1388}) \approx 0.95$

(c) $H_0: \mu = 0$

$H_a: \mu \neq 0$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{19.6 - 0}{20/\sqrt{1388}}$$

(d) $t \sim t_{\frac{\alpha}{2}, n-1}$

(e)