Economics 471: Introductory Econometrics

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University of Alabama

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Fundamentals Exam Key

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider the equation $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2$, where $X_1 \sim (\mu_{X_1}, \sigma^2_{X_1})$ is independent of $X_2 \sim (\mu_{X_2}, \sigma^2_{X_2})$. With this information, answer the following questions:

   (a) What is the expected value of X_1? What is the expected value of X_2?

   (b) What is the variance of X_1? What is the variance of X_2?

   (c) What is the expected value of Y?

   (d) What is the variance of Y?

   (e) What is the marginal effect of X_1 on Y? What is the marginal effect of X_2 on Y?

(a) $E(X_1) = \mu_{X_1}$, $E(X_2) = \mu_{X_2}$

(b) $V(X_1) = \sigma^2_{X_1}$, $V(X_2) = \sigma^2_{X_2}$

(c) $E(Y) = E(\beta_0 + \beta_1 X_1 + \beta_2 X_2)$

$= \beta_0 + \beta_1 \mu_{X_1} + \beta_2 \mu_{X_2}$

(d) $V(Y) = V(\beta_0 + \beta_1 X_1 + \beta_2 X_2)$

$= 0 + \beta_1^2 V(X_1) + \beta_2^2 V(X_2) + 2\beta_1 \beta_2 \operatorname{cov}(X_1, X_2)$

$\overset{\text{indep}}{=} \beta_1^2 \sigma^2_{X_1} + \beta_2^2 \sigma^2_{X_2} + 0$

(e) $\dfrac{\partial Y}{\partial X_1} = \beta_1$, $\dfrac{\partial Y}{\partial X_2} = \beta_2$

2. Consider an independent random sample of data of size n drawn from the continuous distribution of $X \sim (\mu_X, \sigma_X^2)$, i.e., $\{x_i\}_{i=1}^n$. Suppose for whatever reason, we do not like the first observation and we propose the following estimator of μ_X:

$$\widetilde{x} = \frac{x_2 + x_3 + \cdots + x_n}{n}.$$

With this information, answer the following (show your work):

(a) What is the expected value of $\widetilde{x}$?

(b) What is the bias of $\widetilde{x}$?

(c) What is the variance of $\widetilde{x}$?

(d) What is the mean square error of $\widetilde{x}$?

(e) What happens to the results in parts (a-d) when the sample size n tends to infinity? What can be said about this estimator in this scenario?

(a) $E(\widetilde{x}) = E\left(\frac{x_2 + x_3 + \cdots + x_n}{n}\right) = \frac{(n-1)}{n}\mu_X$

(b) $\text{Bias}(\widetilde{x}) = E(\widetilde{x}) - \mu_X$

$= \frac{(n-1)}{n}\mu_X - \mu_X = \frac{(n-1)}{n}\mu_X - \frac{n}{n}\mu_X$

$= \frac{(n-1)-n}{n}\mu_X = -\frac{1}{n}\mu_X$

(c) $V(\widetilde{x}) = V\left(\frac{x_2 + x_3 + \cdots + x_n}{n}\right) = \frac{1}{n^2}V(x_2 + x_3 + \cdots + x_n)$

$\overset{\text{indep't}}{=} \frac{(n-1)}{n^2}\sigma_X^2$

(d) $\text{MSE}(\widetilde{x}) = V(\widetilde{x}) + \text{Bias}(\widetilde{x})^2$

$= \frac{(n-1)}{n^2}\sigma_X^2 + \frac{1}{n^2}\mu_X^2$

(e) as $n \to \infty$ $E(\widetilde{x}) \to \mu_X$, $\text{Bias}(\widetilde{x}) \to 0$

$V(\widetilde{x}) \to 0$ & $\text{MSE}(\widetilde{x}) \to 0 \Rightarrow$

$\widetilde{x}$ is a consistent est of μ_X

3. The sample output from gretl below includes descriptive statistics and a correlation matrix using data from 1388 mothers. The variables listed are the birthweight of their child in ounces (bwght), local cigarette taxes in cents (cigtax), local cigarette price in cents (cigprice) and number of cigarettes smoked daily during pregnancy (cigs).

Summary Statistics, using the observations 1–1388

Variable	Mean	Median	S.D.	Min	Max
bwght	119.0	120.0	20.4	23.0	271.0
cigtax	19.6	20.0	7.8	2.0	38.0
cigprice	131.0	131.0	10.2	104.0	153.0
cigs	2.1	0.0	6.0	0.0	50.0

Correlation coefficients, using the observations 1–1388

bwght	cigtax	cigprice	cigs	
1.0000	0.0478	0.0492	−0.1508	bwght
	1.0000	0.8759	0.0294	cigtax
		1.0000	0.0097	cigprice
			1.0000	cigs

Using the information above, answer the following (give numerical value and formula when feasible):

(a) What is the average cigarette price (cigprice) in dollars?

(b) Draw a potential distribution for the local cigarette taxes (cigtax). Be sure the label the axes as well as the mean and median values.

(c) Draw a potential distribution for the number of cigarettes smoked daily (cigs). Be sure the label the axes as well as the mean and median values.

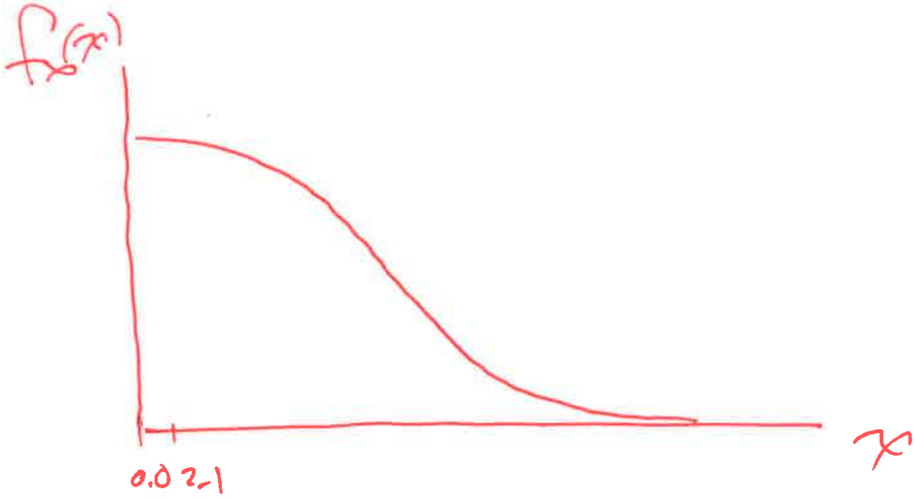
(d) What can be said about the linear dependence between birthweight (bwght) and the number of cigarettes ~~sold daily~~ smoked per day (cigs)?

(e) What is the covariance between local cigarette taxes (cigtax) and number of cigarettes smoked per day (cigs)?

(a) 131 ¢ or $1.31

(b) $f_X(x)$

(c)



(d) $CORR(bwght, cigs) = -0.1508$

negative relationship

(e) $CORR(X, Y) = \dfrac{cov(X, Y)}{sd(x)\, sd(y)}$

$cov(x, y) = sd(x)\, sd(y)\, CORR(x, y)$

$cov(cigtax, cigs) = (7.8)(6.0)(0.0294)$