# Economics 471: Introductory Econometrics

Department of Economics, Finance and Legal Studies

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Fundamentals Exam – Answer Key

1. (a) $E(X) = \int x f_X(x)\,dx = \mu_X = 0$ as stated in the problem

   (b) $V(X) = \int (x-\mu_X)^2 f_X(x)\,dx = \sigma_X^2 = \frac{v}{v-2}$ as stated in the problem

   (c) $\int f_X(x)\,dx = 1$ for any probability density function

   (d) For any standardized random variable $Z = \frac{X-\mu_X}{\sigma_X}$, $E(Z) = 0$ and $V(Z) = 1$

2. (a) $E\left(\overleftrightarrow{x}\right) = E\left(\frac{2}{n}\sum_{i=1}^{n} x_i\right) = \frac{2}{n}\sum_{i=1}^{n} E(x_i) = \frac{2}{n}\sum_{i=1}^{n}\mu_X = \frac{2}{n}n\mu_X = 2\mu_X$

   $E(\widetilde{x}) = E\left(\frac{1}{2n}\sum_{i=1}^{n} x_i\right) = \frac{1}{2n}\sum_{i=1}^{n} E(x_i) = \frac{1}{2n}\sum_{i=1}^{n}\mu_X = \frac{1}{2n}n\mu_X = \frac{1}{2}\mu_X$

   (b) $Bias\left(\overleftrightarrow{x}\right) = E\left(\overleftrightarrow{x}\right) - \mu_x = 2\mu_x - \mu_x = \mu_x$

   $Bias(\widetilde{x}) = E(\widetilde{x}) - \mu_x = \frac{1}{2}\mu_x - \mu_x = -\frac{1}{2}\mu_x$

   (c) $V\left(\overleftrightarrow{x}\right) = V\left(\frac{2}{n}\sum_{i=1}^{n} x_i\right) = \frac{4}{n^2}\sum_{i=1}^{n} V(x_i) = \frac{4}{n^2}\sum_{i=1}^{n}\sigma_X^2 = \frac{4}{n^2}n\sigma_X^2 = \frac{4}{n}\sigma_X^2$

   $V(\widetilde{x}) = V\left(\frac{1}{2n}\sum_{i=1}^{n} x_i\right) = \frac{1}{4n^2}\sum_{i=1}^{n} V(x_i) = \frac{1}{4n^2}\sum_{i=1}^{n}\sigma_X^2 = \frac{1}{4n^2}n\sigma_X^2 = \frac{1}{4n}\sigma_X^2$

   (d) $MSE\left(\overleftrightarrow{x}\right) = Bias\left(\overleftrightarrow{x}\right)^2 + V\left(\overleftrightarrow{x}\right) = \mu_X^2 + \frac{4}{n}\sigma_X^2 \to \mu_X^2 \neq 0$ as $n \to \infty$

   $MSE(\widetilde{x}) = Bias(\widetilde{x})^2 + V(\widetilde{x}) = \frac{1}{4}\mu_X^2 + \frac{1}{4n}\sigma_X^2 \to \frac{1}{4}\mu_X^2 \neq 0$ as $n \to \infty$

3. (a) $\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{312}1982.14$

   (b) $\widehat{\sigma}_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\overline{x})^2 = \frac{1}{311}18521.25$

   (c) $\widehat{\sigma}_{X,Y} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y}) = \frac{1}{311}3517.55$

   (d) $\widehat{\rho}_{X,Y} = \frac{\widehat{\sigma_{X,Y}}}{\widehat{\sigma_X}\widehat{\sigma_Y}} = \frac{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\overline{x})^2}\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i-\overline{y})^2}} = \frac{\frac{1}{311}3517.55}{\sqrt{\frac{1}{311}18521.25}\sqrt{\frac{1}{311}4874.21}}$