Economics 471: Introductory Econometrics

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Fall 2023

Fundamentals Exam Key

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider the table below which gives the value of the sum ($X = Z_1 + Z_2$) of two fair-sided dice (Z_1, Z_2) thrown simultaneously (hint: Z_1 and Z_2 each have equal probability of each event $1, 2, \ldots, 6$ and are independent of one another). With this information, answer the following (*all formulae must be given*):

		Z_1					
		1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
Z_2	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

(a) What is the expected value of the first dice ($E(Z_1)$)?

(b) What is the probability that the sum is 6 ($P(X = 6)$)?

(c) Draw the probability density function of the sum ($f_X(x)$).

(d) What is the probability the sum is 6 given the value of the first dice is a 2 ($P(X = 6|Z_1 = 2)$)?

(e) What is the expected value of the sum given the value of the first dice is a 2 ($E(X|Z_1 = 2)$)?

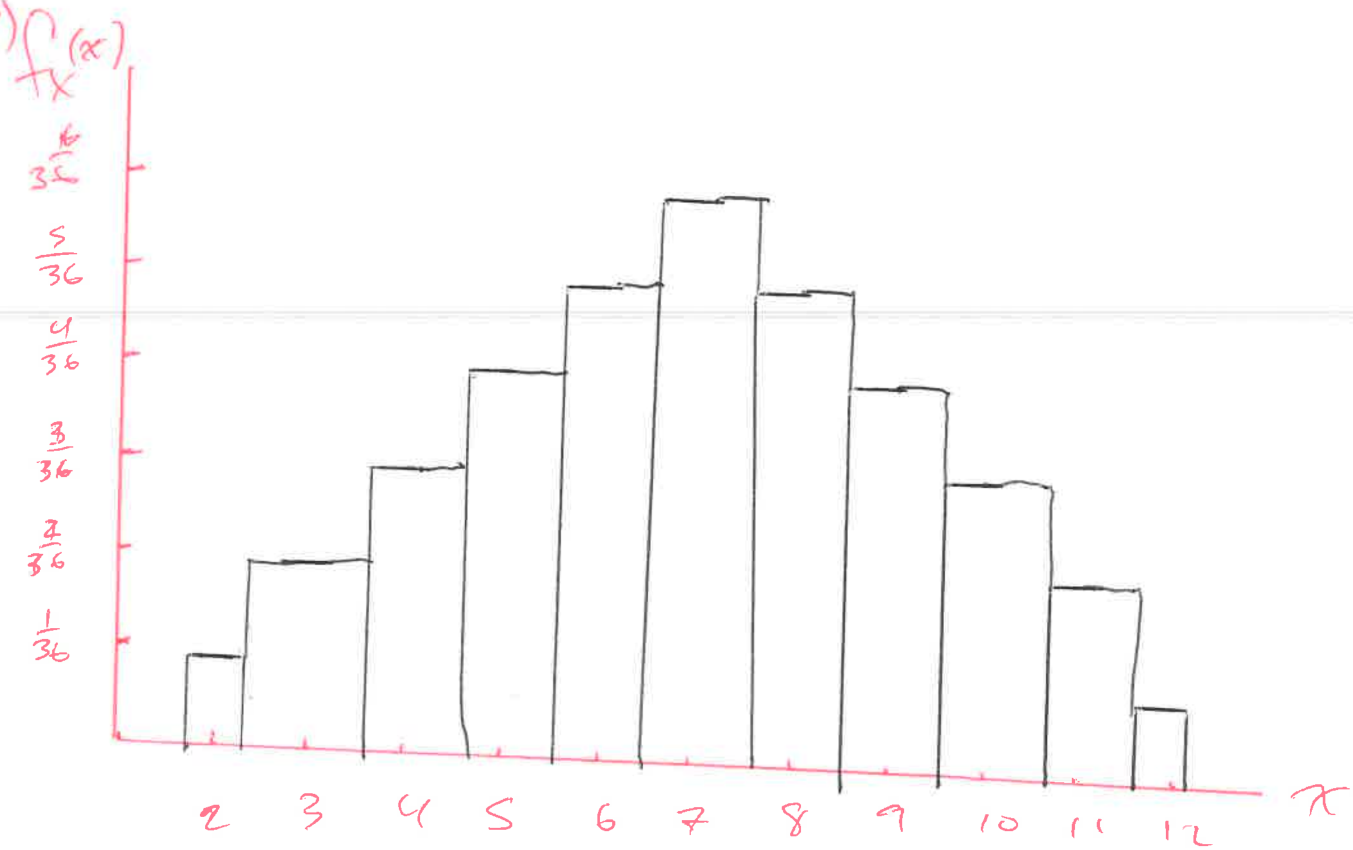
(a) $E(Z_1) = \sum_{j=1}^{6} z_{1j} f_{Z_1}(z_{1j})$

$= (1)(\frac{1}{6}) + (2)(\frac{1}{6}) + (3)(\frac{1}{6}) + (4)(\frac{1}{6}) + (5)(\frac{1}{6}) + 6(\frac{1}{6})$

$= 3\frac{1}{2}$

(b) $P(X=6) = \frac{\#\text{ of } X=6}{\text{total }\#\text{ of possibles}} = \frac{5}{36}$

(c)



(d) $P(X=6 \mid Z_1=2) = \dfrac{\#\text{ of tries } 6}{\#\text{ of possibles}} \mid Z_1 = 2 = \dfrac{1}{6}$

(e) $E(X \mid Z_1=2) = \sum_{j=1}^{6} x_j f_Z(z_1=2)$

$= (3)(\frac{1}{6}) + (4)(\frac{1}{6}) + (5)(\frac{1}{6}) + (6)(\frac{1}{6}) + (7)(\frac{1}{6})$

$= \dfrac{25}{6}$

2. Consider a random variable $X \sim \left(\frac{d_2}{d_2-2}, \frac{2d_2^2(d_1+d_2-2)}{d_1(d_2-2)^2(d_2-4)}\right)$ that comes from the $F-$distribution with d_1 and d_2 degrees of freedom (for the numerator and denominator, respectively). With this information, answer the following (show your work):

(a) Draw a reasonable probability density function for X $(f_X(x))$.

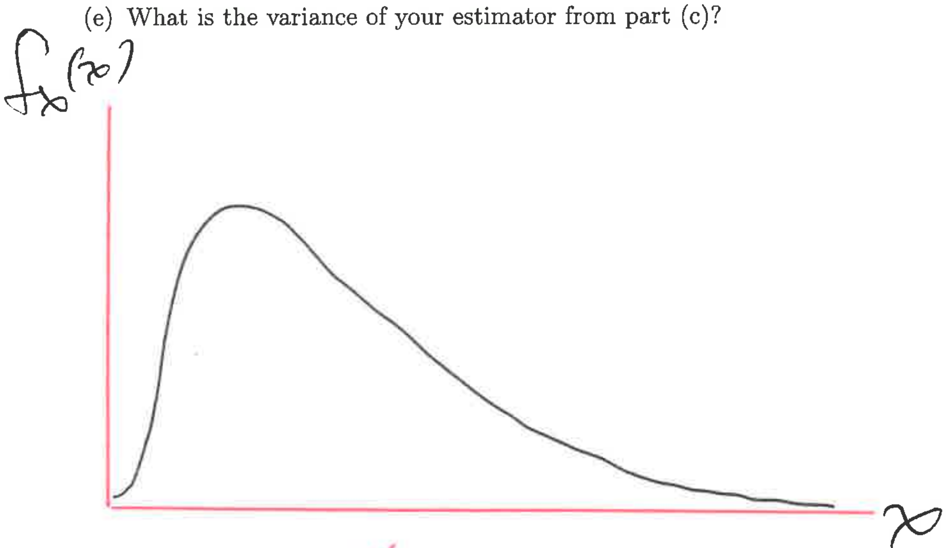
(b) What is the expected value of X $(E(X))$? What is the variance of X $(V(X))$?

(c) Suppose you draw a random sample of data $\{x_i\}_{i=1}^n$ from this distribution. What estimator would you use to estimate the mean (give the formula)?

(d) What is the expected value you estimator from part (c)?

(e) What is the variance of your estimator from part (c)?

(a)



(b) $E(x) = \frac{d_2}{d_2-2}$

$V(x) = \frac{2d_2^2(d_1+d_2-2)}{d_1(d_2-2)^2(d_2-4)}$

(c) $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

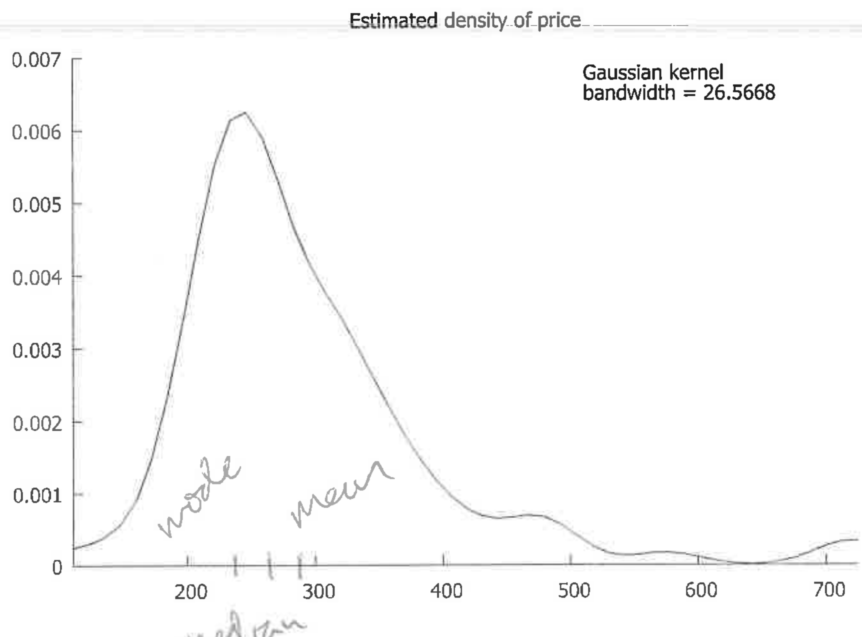
(d) $E(\bar{x}) = E\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(x_i)$

$= \frac{1}{n}\sum_{i=1}^{n} \mu_x = \frac{1}{n} n\mu_x = \mu_x = \frac{d_2}{d_2-2}$

(e) $V(\bar{x}) = V\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) = \frac{1}{n^2} V\left(\sum_{i=1}^{n} x_i\right)$

$\overset{iid}{=} \frac{1}{n^2}\sum_{i=1}^{n} V(x_i) = \frac{1}{n^2}\sum_{i=1}^{n} \sigma_x^2 = \frac{1}{n^2} n\sigma_x^2 = \frac{\sigma_x^2}{n} = \frac{V(x)}{n}$

3. Consider the estimated probablity density function below (price of a home – measured in $1,000s). Using this figure, answer the following:



(a) Draw a reasonable value for the mean. ≈ 293

(b) Draw a reasonable value for the median ≈ 265

(c) Draw a reasonable value for the mode. ≈ 250

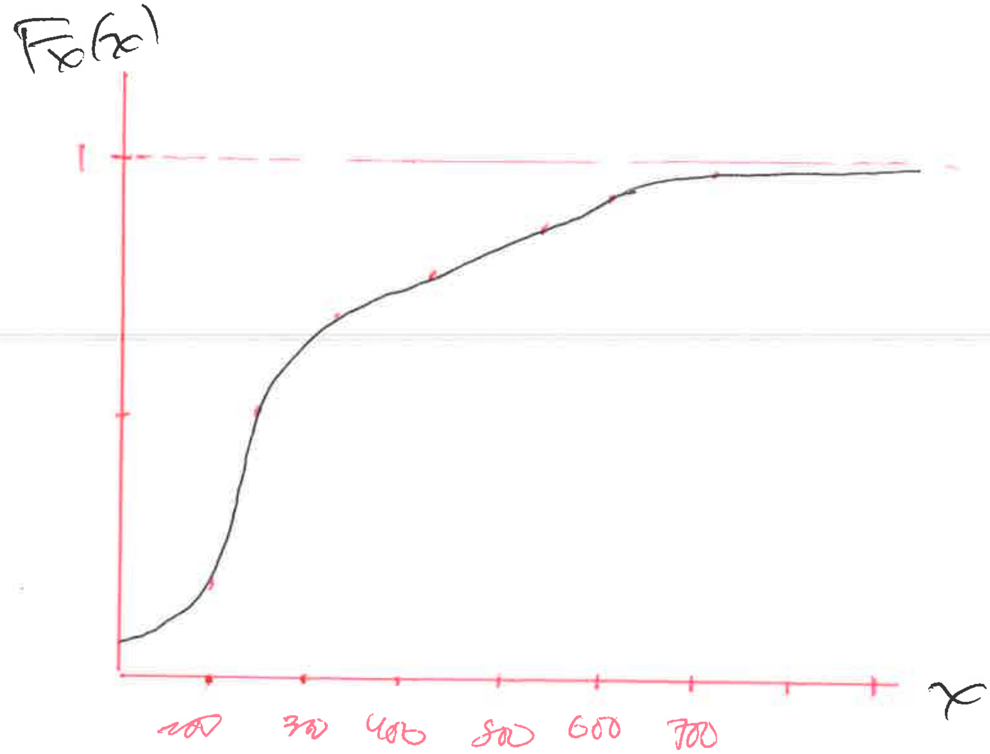
(d) What is the probablity that the price of a home is $250k?

(e) Draw a reasonable cummulative distribution function that corresponds to this estimated probability density function.

(d) $P(X=250) = 0$

prob of any given event for a continuous dist is zero

(c)



half of the housing values are less than the median (≈265)