Economics 471: Introductory Econometrics

Department of Economics, Finance and Legal Studies

University of Alabama

Fall 2019

Fundamentals Exam

Answer Key

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider the discrete random variable X which takes the value 0 with probability 1/2, 1 with probability 1/4 and 2 with probability 1/4. Given this information, answer the following:

   (a) What is the expected value of this random variable?

   (b) What is the variance of this random variable (noting that $V(X) = E(X^2) - E(X)^2$)?

   (c) What is the standard deviation of this random variable?

   (d) Plot the probability density function (x vs $f_X(x)$).

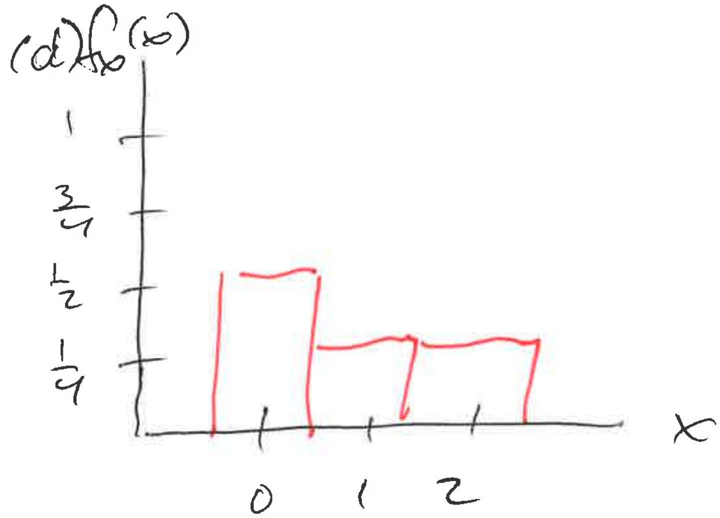
   (e) Plot the cumulative distribution function (x vs $F_X(x)$).

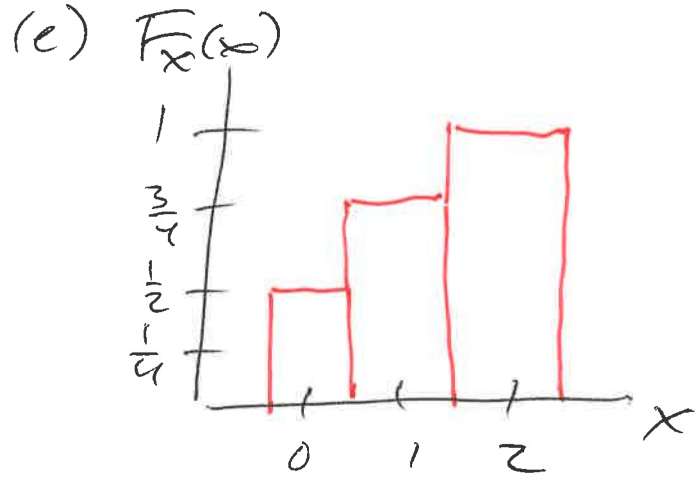
(a) $E(x) = \sum_{j=1}^{3} x_j f_x(x_j) = 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{4} + 2 \cdot \frac{1}{4} = \frac{3}{4}$

(b) $E(x^2) = \sum_{j=1}^{3} x_j^2 f_x(x_j) = 0^2 \cdot \frac{1}{2} + 1^2 \cdot \frac{1}{4} + 2^2 \cdot \frac{1}{4} = \frac{5}{4}$

$V(x) = E(x^2) - E(x)^2 = \frac{5}{4} - \left(\frac{3}{4}\right)^2 = \frac{11}{16}$

(c) $sd(x) = \sqrt{V(x)} = \sqrt{\frac{11}{16}}$





2. Consider the continuous random variables $X \sim (\mu_X, \sigma_X^2)$ and $Y \sim (\mu_Y, \sigma_Y^2)$ with covariance $\sigma_{X,Y} = E\left[(X-\mu_X)(Y-\mu_y)\right]$ and correlation $\rho_{X,Y} = \sigma_{X,Y}/\sigma_X\sigma_Y$. Answer the following:

(a) Show that the covariance can equivalently be written as $\sigma_{X,Y} = E\left[(X-\mu_X)Y\right]$

(b) Suppose that we are interested in the covariance between X and X. Show that this is equivalent to a common measure of dispersion.

(c) Suppose we were interested in the covariance between $(aX+b)$ and $(cY+d)$, where a, b, c and d are all constants greater than zero. Show that this covariance is equal to $ac\sigma_{X,Y}$.

(d) Suppose we were interested in the correlation between $(aX+b)$ and $(cY+d)$, where a, b, c and d are all constants greater than zero. Show that this correlation is equal to $\rho_{X,Y}$.

(e) Suppose that we are interested in the correlation between X and X. Show that this is equal to one.

(a) $\sigma_{X,Y} = E\{[X-\mu_X][Y-\mu_Y]\}$

$= E[(X-\mu_X)Y] - E[(X-\mu_X)\mu_Y]$

$= E[(X-\mu_X)Y] - \mu_Y E(X-\mu_X)$

$= E[(X-\mu_X)Y] - \mu_Y [E(X)-\mu_X]$

$= E[(X-\mu_X)Y] - \mu_Y(\mu_X-\mu_X)$

$= E[(X-\mu_X)Y]$

(b) $\sigma_{X,X} = E[(X-\mu_X)(X-\mu_X)] = E[(X-\mu_X)^2] = \sigma_X^2$

(c) $\text{cov}[(aX+b),(cY+d)] = E\{[aX+b-E(aX+b)]$
$[cY+d-E(cY+d)]\} = E\{[aX+b-aE(X)-b]$
$[cY+d-cE(Y)-d]\} = E\{a[X-E(X)]$
$c[Y-E(Y)]\} = ac\, E[(X-\mu_X)(Y-\mu_Y)]$

$= ac\,\sigma_{X,Y}$

(d) $\text{CORR}\left[(aX+b),(cY+d)\right] = \dfrac{\text{cov}\left[(aX+b),(cY+d)\right]}{sd(aX+b)\,sd(cY+d)}$

$sd(aX+b) = \sqrt{V(aX+b)}$ $\quad sd(cY+d) = \sqrt{V(cY+d)}$

$= \sqrt{a^2 V(X)}$ $\quad = \sqrt{c^2 V(Y)}$

$= |a|\,sd(X)$ $\quad = |c|\,sd(Y)$

$$\text{CORR}\left[(aX+b),(cY+d)\right] = \frac{ac\,\sigma_{X,Y}}{a\,sd(X)\,c\,sd(Y)}$$

$$= \frac{\sigma_{X,Y}}{\sigma_X \sigma_Y}$$

$$= \underline{\rho_{X,Y}}$$

(e) $\text{CORR}(X,X) = \rho_{X,X} = \dfrac{\sigma_{X,X}}{\sigma_X, \sigma_X} = \dfrac{\sigma_X{}^2}{\sigma_X{}^2} = \underline{1}$

3. Consider the relationship between wages (wage), education (educ) and experience (exper). Sample output from gretl includes both the descriptive statistics

Summary Statistics, using the observations 1–526

Variable	Mean	Median	S.D.	Min	Max
wage	5.90	4.65	3.69	0.530	25.0
educ	12.6	12.0	2.77	0.000	18.0
exper	17.0	13.5	13.6	1.00	51.0

and the correlation matrix

Correlation coefficients, using the observations 1–526

wage	educ	exper	
1.0000	0.4059	0.1129	wage
	1.0000	−0.2995	educ
		1.0000	exper

Using this information, answer the following (give numerical value and formula when feasible):

(a) What is the sample size?

(b) What is the sample variance of education?

(c) What is the sample covariance between education and wages?

(d) What is the sample correlation between education and wages?

(e) Plot a feasible probability density function for experience (be sure to list the mean and median values on the plot).

(a) $n = \underline{526}$

(b) $\hat{\sigma}^2_{edu} = \frac{1}{n-1}\sum_{i=1}^{526}(educ_i - \overline{educ})^2$

$= (sd(educ))^2 = \underline{2.77^2}$

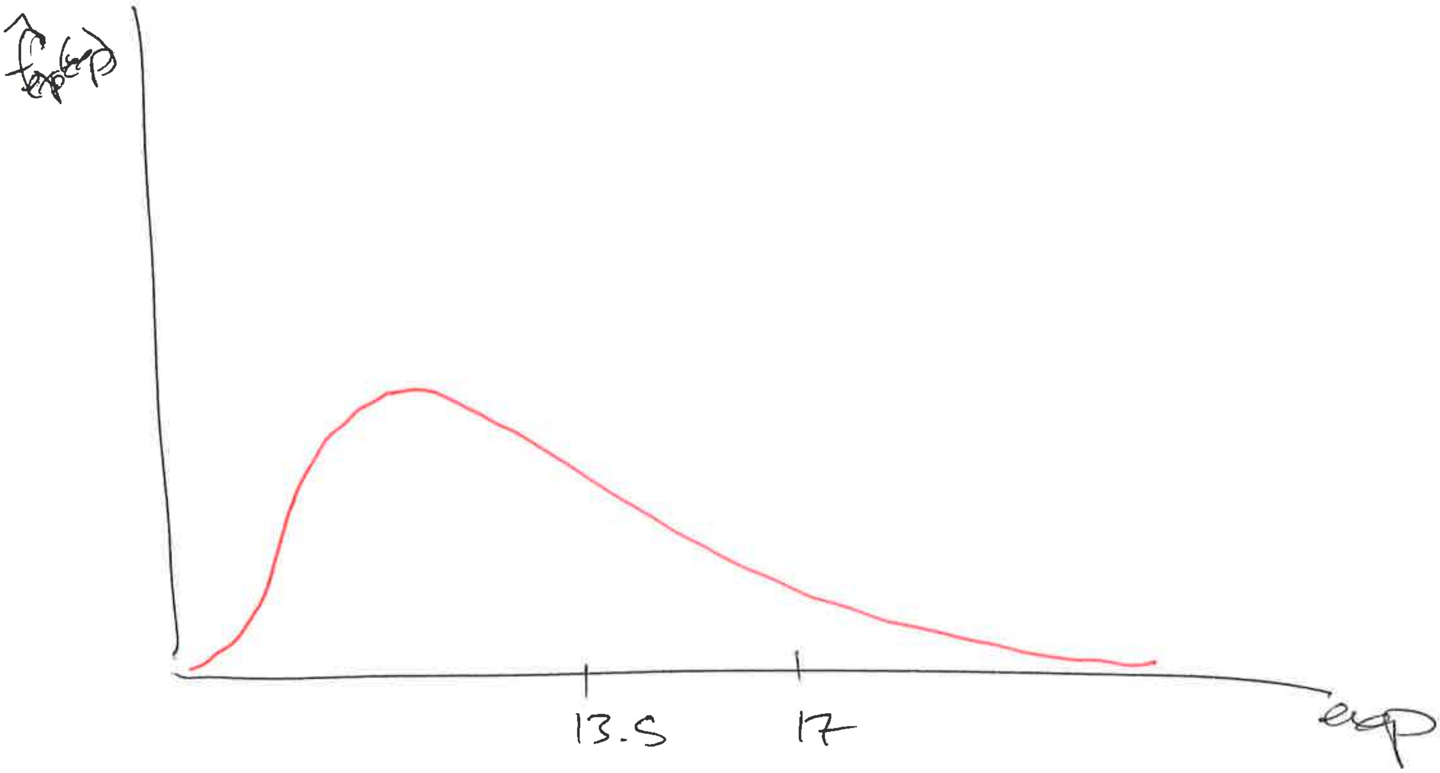
(c) $\hat{\sigma}_{edu,inc} = \frac{1}{n-1}\sum_{i=1}^{526}(educ_i - \overline{educ})(wage_i - \overline{wage})$

$\hat{\rho}_{educ,wage} = \frac{\hat{\sigma}_{educ,wage}}{sd(educ)\,sd(wage)} \Rightarrow$

$\underline{(0.4059)(2.77)(3.69)} = \hat{\sigma}_{educ,wage}$

(d) see above $\underline{0.4059}$

(e)



positive skew