# Economics 471: Introductory Econometrics

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Fundamentals Exam Key

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider the rules for summations we showed in class. With this information, show the following results (show your work):

   (a) $\sum_{i=1}^{n}(x_i - \bar{x}) = 0$

   (b) $\sum_{i=1}^{n}(x_i - \bar{x})^2 = (\sum_{i=1}^{n} x_i^2) - n\bar{x}^2$

   (c) $\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n}(x_i - \bar{x})y_i$

   (d) $\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n} x_i(y_i - \bar{y})$

   (e) $\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = (\sum_{i=1}^{n} x_i y_i) - n\bar{x}\bar{y}$

(a) $\sum_{i=1}^{n}(x_i - \bar{x}) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \bar{x} = n\bar{x} - n\bar{x} = 0$

(b) $\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i\bar{x} - \sum_{i=1}^{n} \bar{x}x_i + \sum_{i=1}^{n} \bar{x}^2 = \sum_{i=1}^{n} x_i^2 - 2\bar{x}\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \bar{x}^2$

$= \sum_{i=1}^{n} x_i^2 - 2n\bar{x}^2 + n\bar{x}^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$

(c) $\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n}(x_i - \bar{x})y_i - \sum_{i=1}^{n}(x_i - \bar{x})\bar{y} = \sum_{i=1}^{n}(x_i - \bar{x})y_i - \bar{y}\sum_{i=1}^{n}(x_i - \bar{x})$

$= \sum_{i=1}^{n}(x_i - \bar{x})y_i$

(d) $\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n} x_i(y_i - \bar{y}) - \sum_{i=1}^{n} \bar{x}(y_i - \bar{y}) = \sum_{i=1}^{n} x_i(y_i - \bar{y})$

$- \bar{x}\sum_{i=1}^{n}(y_i - \bar{y}) = \sum_{i=1}^{n} x_i(y_i - \bar{y})$

(e) $\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i\bar{y} - \sum_{i=1}^{n} \bar{x}y_i + \sum_{i=1}^{n} \bar{x}\bar{y}$

$= \sum_{i=1}^{n} x_i y_i - \bar{y}\sum_{i=1}^{n} x_i - \bar{x}\sum_{i=1}^{n} y_i + n\bar{x}\bar{y}$

$= \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y} - n\bar{x}\bar{y} + n\bar{x}\bar{y}$

$= \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}$

2. Consider the continuous random variable $X$ with probability density function (pdf)

$$f_X(x) = \begin{cases} \frac{1}{2}, & \text{if } -1 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

which is better known as a Uniform distribution from $-1$ to $1$. With this information, answer the following:

(a) Draw this pdf.

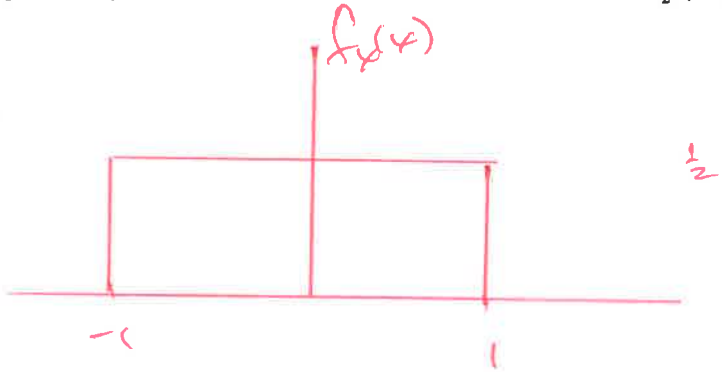
(b) What is the mean (expected value) of this random variable? What is the median value of this random variable?

(c) What is the mode of this random variable?

(d) Show that the area under this pdf is equal to 1.

(e) Find the probability that the random variable is less than or equal to $\frac{1}{2}$ (i.e., $P\left(X \leq \frac{1}{2}\right)$).

(a)




(b) $E(x) = \int x f_X(x) dx = 0$ median $= 0$

(c) every value from $-1$ to $1$

(d) area $= L \times W = (2)(\frac{1}{2}) = 1$

(e) $P(x \leq \frac{1}{2}) = (\frac{3}{2})(\frac{1}{2}) = \frac{3}{4}$

3. The sample output from gretl below includes descriptive statistics for wages (wage), education (educ) and experience (exper).

Summary Statistics, using the observations 1–526

| Variable | Mean | Median | S.D. | Min | Max |
|---|---|---|---|---|---|
| wage | 5.90 | 4.65 | 3.69 | 0.530 | 25.0 |
| educ | 12.60 | 12.00 | 2.77 | 0.000 | 18.0 |
| exper | 17.00 | 13.50 | 13.60 | 1.000 | 51.0 |

Our primary concern in this question is the (unknown) population mean for wages. Using the information above, answer the following (give numerical value and formula when feasible):

(a) Suppose we want to test the null that the true mean wage is \$6 (i.e., $H_0 : \mu_{wage} = 6$ vs. the alternative $H_1 : \mu_{wage} \neq 6$). Construct the test statistic for this null hypothesis (i.e., construct the $t$−statistic).

(b) For the result in part (a), draw the null distribution for the test statistic and place a 'reasonable' value for the test statistic on the figure (hint: $\sqrt{526} \approx 23$).

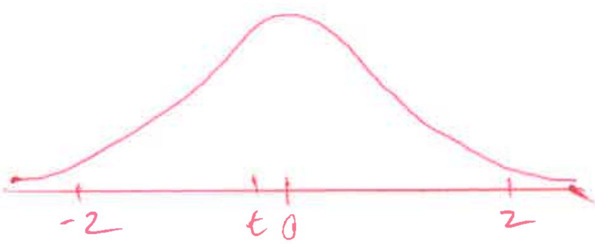
(c) Using the rule-of-thumb critical value from class, what would you expect the conclusion of the test from part (a) to be? Why?

(d) Re-draw the figure from part (b). Show the p-value graphically on this figure (i.e., shade the relevant area).

(e) Suppose we want to test the null that the true mean wage is \$0 (i.e., $H_0 : \mu_{wage} = 0$ vs the alternative $H_1 : \mu_{wage} \neq 0$). Draw the null distribution for the test statistic and show a 'reasonable' p-value.

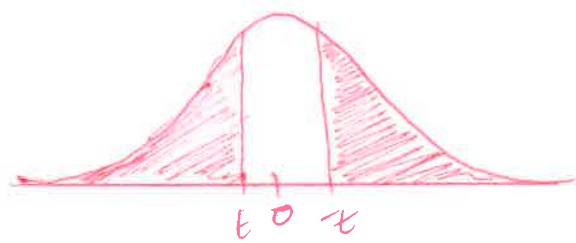
(a) $t = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{5.90 - 6.00}{3.69/\sqrt{526}}$

(b)




(c) fail to reject $H_0$ as $|t| < 2$

(d)




(e) $t = \dfrac{5.90 - 0}{3.69/\sqrt{526}}$