Economics 471: Econometrics

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Final Exam – Answers

1. (a) $E(ux) = 0 \Rightarrow \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \widehat{\beta}x_i\right)x_i = 0 \Rightarrow \widehat{\beta} = \sum_{i=1}^{n} y_i x_i / \sum_{i=1}^{n} x_i^2$

   (b) $\sum_{i=1}^{n} \widehat{u}_i^2 = \sum_{i=1}^{n}\left(y_i - \widehat{\beta}x_i\right)^2, \partial/\partial\widehat{\beta} = -2\sum_{i=1}^{n}\left(y_i - \widehat{\beta}x_i\right)x_i \Rightarrow \widehat{\beta} = \sum_{i=1}^{n} y_i x_i / \sum_{i=1}^{n} x_i^2$

   (c) $E\left(\widehat{\beta}\right) = E\left(\sum_{i=1}^{n} y_i x_i / \sum_{i=1}^{n} x_i^2\right) = E\left[\sum_{i=1}^{n}(\beta x_i + u_i)x_i / \sum_{i=1}^{n} x_i^2\right] = \beta + E\left(\sum_{i=1}^{n} u_i x_i / \sum_{i=1}^{n} x_i^2\right) = \beta$

   (d) $E\left(\widehat{\beta}\right) = E\left(\sum_{i=1}^{n} y_i x_i / \sum_{i=1}^{n} x_i^2\right) = E\left[\sum_{i=1}^{n}(\alpha + \beta x_i + u_i)x_i / \sum_{i=1}^{n} x_i^2\right] = \alpha\sum_{i=1}^{n} x_i / \sum_{i=1}^{n} x_i^2 + \beta + E\left(\sum_{i=1}^{n} u_i x_i / \sum_{i=1}^{n} x_i^2\right) = \beta + \alpha\sum_{i=1}^{n} x_i / \sum_{i=1}^{n} x_i^2$ $Bias\left(\widehat{\beta}\right) = E\left(\widehat{\beta}\right) - \beta = \alpha\sum_{i=1}^{n} x_i / \sum_{i=1}^{n} x_i^2$

   (e) $\widehat{\beta} = \sum_{i=1}^{n}(y_i - \overline{y})(x_i - \overline{x}) / \sum_{i=1}^{n}(x_i - \overline{x})^2$

2. (a) $E(u) = 0 \Rightarrow \frac{1}{n}\sum_{i=1}^{n}(y_i - \widehat{\alpha}) = 0 \Rightarrow \widehat{\alpha} = \frac{1}{n}\sum_{i=1}^{n} y_i$

   (b) $\sum_{i=1}^{n} \widehat{u}_i^2 = \sum_{i=1}^{n}(y_i - \widehat{\alpha})^2, \partial/\partial\widehat{\alpha} = -2\sum_{i=1}^{n}(y_i - \widehat{\alpha}) \Rightarrow \widehat{\alpha} = \frac{1}{n}\sum_{i=1}^{n} y_i$

   (c) $V(\widehat{\alpha}) = V\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right) = \frac{\sigma^2}{n}$

      i. $n \uparrow \Rightarrow V(\widehat{\alpha}) \downarrow$

      ii. $\sigma^2 \uparrow \Rightarrow V(\widehat{\alpha}) \uparrow$

      iii. Plays no role

      iv. Plays no role

   (d) $H_0 : \beta_1 = \cdots = \beta_k = 0$, but we have no regressors in this model and so there is nothing to test

3. (a) If the woman has zero years of education, the probability she works is equal to -0.146. This is not reasonable as the probability is negative.

   (b) If a woman increases her education by one year, the probability she works goes up by 0.038. The sign is reasonable and the magnitude is up for debate.

   (c) $t = -0.146/0.121 = -1.207 < 2 \Rightarrow$ fail to reject, $t = 0.038/0.014 = 2.714 > 2 \Rightarrow$ reject null

   (d) Figure: intercept is at -0.146, intersects x axis at 3.84 and reaches value 1 at 22.47 (both to be shown in part (e)).

   (e) $0 = -0.146 + 0.038x^{*} \Rightarrow x^{*} = 0.146/0.038 = 3.84$, $1 = -0.416 + 0.038x^{**} \Rightarrow x^{**} = (1 - 0.146)/0.038 = 22.47$

4. (a) $\sum_{i=1}^{n}\widehat{e}_i = \sum_{i=1}^{n} y_i - \widehat{\Lambda}_0 - \widehat{\Lambda}_1 x_{1i} - \widehat{\Lambda}_2 x_{2i} - \cdots - \widehat{\Lambda}_k x_{ki} = \sum_{i=1}^{n} y_i - \left(\overline{y} - \widehat{\Lambda}_1\overline{x}_1 - \widehat{\Lambda}_2\overline{x}_2 - \cdots - \widehat{\Lambda}_k\overline{x}_k\right) - \widehat{\Lambda}_1 x_{1i} - \widehat{\Lambda}_2 x_{2i} - \cdots - \widehat{\Lambda}_k x_{ki} = n\overline{y} - n\overline{y} + \widehat{\Lambda}_1\overline{x}_1 + \widehat{\Lambda}_2\overline{x}_2 + \cdots + \widehat{\Lambda}_k\overline{x}_k - \widehat{\Lambda}_1\overline{x}_1 - \widehat{\Lambda}_2\overline{x}_2 - \cdots - \widehat{\Lambda}_k\overline{x}_k = 0$

(b) $\sum_{i=1}^{n} \widehat{e}_i x_{1i} = \sum_{i=1}^{n} \left(y_i - \widehat{\Lambda}_0 - \widehat{\Lambda}_1 x_{1i}\right) x_{1i} = \sum_{i=1}^{n} \left[y_i - \left(\overline{y} - \widehat{\Lambda}_1 \overline{x}_1\right) - \widehat{\Lambda}_1 x_{1i}\right] x_{1i} = \sum_{i=1}^{n} \left[(y_i - \overline{y}) - \widehat{\Lambda}_1 (x_{1i} - \overline{x}_1)\right] x_{1i} = \sum_{i=1}^{n} \left[(y_i - \overline{y}) - \widehat{\Lambda}_1 (x_{1i} - \overline{x}_1)\right] (x_{1i} - \overline{x}_1) = \sum_{i=1}^{n} \left[(y_i - \overline{y})(x_{1i} - \overline{x}_1) - \widehat{\Lambda}_1 (x_{1i} - \overline{x}_1)^2\right] = \sum_{i=1}^{n} (y_i - \overline{y})(x_{1i} - \overline{x}_1) - \widehat{\Lambda}_1 \sum_{i=1}^{n} (x_{1i} - \overline{x}_1)^2 = \sum_{i=1}^{n} (y_i - \overline{y})(x_{1i} - \overline{x}_1) - \frac{\sum_{i=1}^{n} (y_i - \overline{y})(x_{1i} - \overline{x}_1)}{\sum_{i=1}^{n} (x_{1i} - \overline{x}_1)^2} \sum_{i=1}^{n} (x_{1i} - \overline{x}_1)^2 = \sum_{i=1}^{n} (y_i - \overline{y})(x_{1i} - \overline{x}_1) - \sum_{i=1}^{n} (y_i - \overline{y})(x_{1i} - \overline{x}_1) = 0$

(c) $\sum_{i=1}^{n} \widehat{e}_i x_{2i} = \sum_{i=1}^{n} \left(y_i - \widehat{\Lambda}_0 - \widehat{\Lambda}_2 x_{2i}\right) x_{2i} = \sum_{i=1}^{n} \left[y_i - \left(\overline{y} - \widehat{\Lambda}_2 \overline{x}_2\right) - \widehat{\Lambda}_2 x_{2i}\right] x_{2i} = \sum_{i=1}^{n} \left[(y_i - \overline{y}) - \widehat{\Lambda}_2 (x_{2i} - \overline{x}_2)\right] x_{2i} = \sum_{i=1}^{n} \left[(y_i - \overline{y}) - \widehat{\Lambda}_2 (x_{2i} - \overline{x}_2)\right] (x_{2i} - \overline{x}_2) = \sum_{i=1}^{n} \left[(y_i - \overline{y})(x_{2i} - \overline{x}_2) - \widehat{\Lambda}_2 (x_{2i} - \overline{x}_2)^2\right] = \sum_{i=1}^{n} (y_i - \overline{y})(x_{2i} - \overline{x}_2) - \widehat{\Lambda}_2 \sum_{i=1}^{n} (x_{2i} - \overline{x}_2)^2 = \sum_{i=1}^{n} (y_i - \overline{y})(x_{2i} - \overline{x}_2) - \frac{\sum_{i=1}^{n} (y_i - \overline{y})(x_{2i} - \overline{x}_2)}{\sum_{i=1}^{n} (x_{2i} - \overline{x}_2)^2} \sum_{i=1}^{n} (x_{2i} - \overline{x}_2)^2 = \sum_{i=1}^{n} (y_i - \overline{y})(x_{2i} - \overline{x}_2) - \sum_{i=1}^{n} (y_i - \overline{y})(x_{2i} - \overline{x}_2) = 0$

(d) $\widehat{y}_i = \widehat{\Lambda}_0 + \widehat{\Lambda}_1 x_{1i} + \widehat{\Lambda}_2 x_{2i} + \cdots + \widehat{\Lambda}_k x_{ki}$, evaluated at $(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k)$ leads to $\widehat{\Lambda}_0 + \widehat{\Lambda}_1 \overline{x}_1 + \widehat{\Lambda}_2 \overline{x}_2 + \cdots + \widehat{\Lambda}_k \overline{x}_k = \left(\overline{y} - \widehat{\Lambda}_1 \overline{x}_1 - \widehat{\Lambda}_2 \overline{x}_2 - \cdots - \widehat{\Lambda}_k \overline{x}_k\right) + \widehat{\Lambda}_1 \overline{x}_1 + \widehat{\Lambda}_2 \overline{x}_2 + \cdots + \widehat{\Lambda}_k \overline{x}_k = \overline{y}$