Economics 471: Econometrics

Department of Economics, Finance and Legal Studies

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Final Exam

The exam consists of four questions on four pages. Each question is of equal value.

1. Consider the model $y_i = \beta x_i + u_i$, $i = 1, 2, \ldots, n$.

   (a) Using the method of moments estimator, derive the estimate of β.

   (b) Using the least-squares estimator, derive the estimate of β.

   (c) Show that the estimator derived in part (b) is unbiased? (Hint: show that the expected value of the estimator is equal to β)

   (d) Suppose the true data generating process is $y_i = \alpha + \beta x_i + u_i$. Derive the expected value of the estimator from the model in part (b). Show that this estimator of β is biased and state the bias.

   (e) Define (do not derive) an unbiased estimator of the slope coefficient β when the true model is $y_i = \alpha + \beta x_i + u_i$.

2. Consider the model $y_i = \alpha + u_i$, $i = 1, 2, \ldots, n$.

   (a) Using the method of moments estimator, derive the estimate of α.

   (b) Using the least-squares estimator, derive the estimate of α.

   (c) For the estimator derived in part (b), derive the variance.

   (d) Explain what happens to the estimator variance in part (c) when:

      i. The sample size (n) increases.

      ii. The error variance (σ^2) increases.

      iii. The variation in x ($\sum_{i=1}^{n} (x_i - \overline{x})^2$) increases.

      iv. The correlation amongst the regressors (R_x^2) increases.

   (e) Give the null hypothesis for validity of the regression. What is the major issue with employing this test for this model?

3. Consider the following regression with respect to female labor participation (y) on years of education (x):

$$y_i = \alpha + \beta x_i + u_i,\ i = 1, 2, \ldots, n$$

where y is binary (0 or 1, 1 signifying that the woman chooses to work) and x is measured in years (≥ 0). Assume that after OLS, we obtain the parameters of the model as

$$y_i = \underset{(0.121)}{-0.146} + \underset{(0.014)}{0.038} x_i + u_i$$

where the number below the estimated coefficient in parentheses is the standard error of the estimate. With the above information, answer the following:

(a) Interpret the estimated coefficient for α. Does this seem reasonable?

(b) Interpret the estimated coefficient for β. Does this seem reasonable?

(c) Test the null that the coefficient in part (a) is zero. Test the null that the coefficient in part (b) is zero.

(d) Draw a figure with probability of labor force participation on the vertical axis and years of education on the horizontal axis. Draw and label the regression line.

(e) How many years of education does it take before labor force participation is possible (positive probability)? How many year of education does it take for labor force participation to be imminent?

4. Consider the residuals obtained via an OLS regression of y on x_1, x_2, ... , x_k, i.e. $y_i = \Lambda_0 + \Lambda_1 x_{1i} + \Lambda_2 x_{2i} + \cdots + \Lambda_k x_{ki} + e_i$, where e_i is the error term for observation i, $i = 1, 2, \ldots n$. Show the following:

   (a) $\sum_{i=1}^{n} \widehat{e}_i = 0$

   (b) $\sum_{i=1}^{n} \widehat{e}_i x_{1i} = 0$, under the assumption that $\Lambda_2 = \Lambda_3 = \cdots = \Lambda_k = 0$

   (c) $\sum_{i=1}^{n} \widehat{e}_i x_{2i} = 0$, under the assumption that $\Lambda_1 = \Lambda_3 = \Lambda_4 = \cdots = \Lambda_k = 0$

   (d) Show that $(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_k, \overline{y})$ is on the regression line