Economics 471: Econometrics

Department of Economics, Finance and Legal Studies
University of Alabama
Fall 2013

Final Exam – Answers

1. (a) $\widehat{\beta} = \frac{\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}{\sum_{i=1}^{n}(x_i-\overline{x})^2} = \frac{3446.226}{19122.32} = 0.1802201$, $\widehat{\alpha} = \overline{y} - \widehat{\beta}\overline{x} = \frac{1945.26}{274} - 0.1802201\frac{1774.00}{274} = 5.93266$

   (b) The estimate 0.1802201 of $\widehat{\beta}$ means that an increase in firm tenure x_i of 1 year is associated on average with an increase in male employee's hourly wage rate equal to 0.18022 dollars per hour.

   (c) $\widehat{\sigma}^2 = \frac{1}{N-2}\sum_{i=1}^{n}\widehat{u}_i^2 = \frac{1}{272}4105.297 = 15.093$

   (d) $Var\left(\widehat{\beta}\right) = \frac{\widehat{\sigma}^2}{\sum_{i=1}^{n}(x_i-\overline{x})^2} = \frac{15.093}{19122.32} = 0.00078929$

   (e) $R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{\sum_{i=1}^{n}\widehat{u}_i^2}{\sum_{i=1}^{n}(y_i-\overline{y})^2} = 1 - \frac{4105.297}{4726.377} = 1 - 0.8686 = 0.1314$. The value of 0.1314 indicates that 13.14 percent of the total sample (or observed) variation in y_i (employees' hourly wage rates) is attributed to, or explained by, the sample regression function of the regressor x_i (firm tenure).

   (f) $t_{\widehat{\beta}} = \frac{\widehat{\beta}-0}{se\left(\widehat{\beta}\right)} = \frac{0.18022-0}{\sqrt{15.093}} = \frac{0.18022}{0.0280943} = 6.4148$. Since $6.4148 > 2$ then we reject the null that the true value of the coefficient is equal to zero.

2. (a) Assumptions 1, 2 and 3 are violated - the model is incorrectly specified and hence, $E(u) \neq 0$ and $E(u|x) \neq 0$

   (b) $\widehat{\beta} = \frac{\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}{\sum_{i=1}^{n}(x_i-\overline{x})^2}$

   (c) $E\left(\widehat{\beta}\right) = E\left(\frac{\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}{\sum_{i=1}^{n}(x_i-\overline{x})^2}\right) = E\left(\frac{\sum_{i=1}^{n}(x_i-\overline{x})\left(\alpha+\beta x_i+\gamma x_i^2+u_i\right)}{\sum_{i=1}^{n}(x_i-\overline{x})^2}\right) = \beta + \frac{\sum_{i=1}^{n}(x_i-\overline{x})\left(\gamma x_i^2\right)}{\sum_{i=1}^{n}(x_i-\overline{x})^2}$

   (d) $Bias\left(\widehat{\beta}\right) = E\left(\widehat{\beta}\right) - \beta = \gamma\frac{\sum_{i=1}^{n}(x_i-\overline{x})x_i^2}{\sum_{i=1}^{n}(x_i-\overline{x})^2}$

   (e) $CORR\left(x_i, x_i^2\right) > 0$ and $\gamma > 0$. Thus, the sign of the bias is positive.

   (f) This is the correct model, thus the expected value of $\widehat{\beta}$ will be equal to β

3. (a) $\frac{\partial y}{\partial x_1} = \beta_1 + \beta_{12}x_2 + 2\beta_{11}x_1$; $\frac{\partial y}{\partial x_2} = \beta_2 + \beta_{12}x_1 + 2\beta_{22}x_2$

   (b) $y = \alpha+\beta_1x_1+\beta_2x_2+\beta_{12}x_1x_2+\beta_{11}x_1^2+\beta_2x_2^2+\delta_0D+\delta_1x_1D+\delta_2x_2D+\delta_{12}x_1x_2D+\delta_{11}x_1^2D+\delta_{22}x_2^2D+u$

   (c) We have two groups: for group 1 $D = 1$ for group 2 $D = 0$, therefore the partial effects for group 1 for each x are $\frac{\partial y}{\partial x_1} = \beta_1 + \beta_{12}x_2 + 2\beta_{11}x_1 + \delta_1 + \delta_{12}x_2 + 2\delta_{11}x_1$; $\frac{\partial y}{\partial x_2} = \beta_2 + \beta_{12}x_1 + 2\beta_{22}x_2 + \delta_2 + \delta_{12}x_1 + 2\delta_{22}x_2$. For group 2 we assume that $D = 0$ and therefore they are the same as in part (a)

(d) For the chow test we assume that the slopes are the same for each group. Consider the three separate regressions for the pooled group, group 1 and group 2 as

$$\begin{aligned} y &= \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \beta_2 x_2^2 + u \\ y &= \gamma_0 + \gamma_1 x_1 + \gamma_2 x_2 + \gamma_{12} x_1 x_2 + \gamma_{11} x_1^2 + \gamma_2 x_2^2 + u \\ y &= \delta_0 + \delta_1 x_1 + \delta_2 x_2 + \delta_{12} x_1 x_2 + \delta_{11} x_1^2 + \delta_2 x_2^2 + u \end{aligned}$$

Therefore our null hypothesis is $H_0 : \gamma_0 = \delta_0,\ \gamma_1 = \delta_1,\ \gamma_2 = \delta_2,\ \gamma_{12} = \delta_{12},\ \gamma_{11} = \delta_{11},\ \gamma_{22} = \delta_{22}$. The F-statistic for this problem is

$$F = \frac{\left(SSR_p - \left(SSR_1 + SSR_2\right)\right)/6}{\left(SSR_1 + SSR_2\right)/\left(n - 2\left(5+1\right)\right)}$$

4. (a) Below-average male intercept: $1.869431 - 0.184294$, Above-average male intercept: 1.869431, Below-average female intercept: $1.869431 - 0.547765 - 0.184294 + 0.033818$, Above-average female intercept: $1.869431 - 0.547765$

(b) A rejection of the null would imply that *men* with below average looks earn different wages than *men* with above average looks?

(c) The t-test statistic for this null is -16.22105 with a p-value of 0.0000. Thus we reject the null that the two groups make the same wages.

(d) The F-test statistic for this null is 0.116428 with a p-value of 0.950490. Thus we fail to reject the null of homoskedasticity.

(e) In the fitted values formulation we regress the squared residuals on the fitted values and the squares of the fitted values. Specifically, the equation is

$$\widehat{u}_i^2 = \gamma_0 + \gamma_1 \widehat{y}_i + \gamma_2 \widehat{y}_i^2 + v_i$$

and the null is $H_0 : \gamma_1 = \gamma_2 = 0$. We implement this as a standard F-test where the restricted model is only an intercept. Failure to reject this null would state that the squared residuals are not a function of the x's and thus the error is homoskedastic.