Economics 471: Econometrics

Department of Economics, Finance and Legal Studies

University of Alabama

Fall 2013

Final Exam

The exam consists of four questions on four pages. Each question is of equal value.

1. A researcher is using data for a sample of 274 male employees to investigate the relationship between hourly wage rates y_i (measured in dollars per hour) and firm tenure x_i (measured in years). Preliminary analysis of the sample data produces the following sample ($n = 274$) information:

$$\sum_{i=1}^{n} y_i = 1945.26 \qquad \sum_{i=1}^{n} x_i = 1774.00 \qquad \sum_{i=1}^{n} y_i^2 = 18536.73$$
$$\sum_{i=1}^{n} x_i^2 = 30608.00 \qquad \sum_{i=1}^{n} x_i y_i = 16040.72 \qquad \sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y}) = 3446.226$$
$$\sum_{i=1}^{n} (y_i - \overline{y})^2 = 4726.377 \qquad \sum_{i=1}^{n} (x_i - \overline{x})^2 = 19122.32 \qquad \sum_{i=1}^{n} \widehat{u}_i^2 = 4105.297$$

Use the above sample information to answer all the following questions. Answer all questions assuming the simple linear regression model $y_i = \alpha + \beta x_i + u_i$. Show explicitly all formulas and calculations.

   (a) Use the above information to compute OLS estimates of the intercept coefficient α and the slope coefficient β.

   (b) Interpret the slope coefficient estimate you calculated in part (a). In other words, explain what the numeric value you calculated for β means.

   (c) Calculate an estimate of σ^2, the error variance.

   (d) Calculate an estimate of $Var\left(\widehat{\beta}\right)$, the variance of $\widehat{\beta}$.

   (e) Compute the value of R^2, the coefficient of determination for the estimated OLS sample regression equation. Briefly explain what the calculated value of R^2 means.

   (f) Calculate the sample value of the $t$statistic for testing the null hypothesis $H_0 : \beta = 0$ against the alternative hypothesis $H_1 : \beta \neq 0$. What do you conclude from this test?

2. Consider the case where the true data generating process is $y_i = \alpha + \beta x_i + \gamma x_i^2 + u_i$. Assume that you instead run the model $y_i = \alpha + \beta x_i + \varepsilon_i$.

   (a) What Gauss-Markov assumption(s) are violated in your model?

   (b) Give the estimator of β for the model.

   (c) What is $E\left(\widehat{\beta}\right)$ from the model?

   (d) Give the equation for the bias.

   (e) Suppose $x_i > 1\ \forall i$ and $\gamma > 0$. What is the sign of the bias?

(f) Suppose you next run the model as $y_i = \alpha + \beta x_i + \gamma x_i^2 + u_i$. What is $E\left(\widehat{\beta}\right)$ in this model (just state, do not derive)?

2. Consider the following regression

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \beta_2 x_2^2 + u$$

(a) Give the partial effect of y given x_1. Give the partial effect of y given x_2.

(b) Suppose there are two groups of individuals, say male and female. Write an equation that allows for both an intercept shift and different slope parameters?

(c) For the model in part (b), give the partial effect of y given x_1 for each group separately. Do the same thing for the partial effect of y given x_2.

(d) Suppose you were interested in testing that the two groups are the same. Using the Chow Test, write the null hypothesis and the F-statistic for this particular scenario.

3. Economists have investigated the relationship between an individual's appearance and their earnings. Appearance is measured as above or below average looks. Define the variable representing "below average looks" as

$$belavg = \begin{cases} 1 \text{ if below average looks} \\ 0 \text{ otherwise} \end{cases}$$

The model, allowing for different parameters for males and females is

$$\ln(wage) = a_0 + a_1 belavg + a_2 female + a_3 belavg * female + v$$

where

$$female = \begin{cases} 1 \text{ if female} \\ 0 \text{ otherwise} \end{cases}$$

Given this information and the attached tables, answer the following questions:

(a) Table 1 gives the estimates of the model using a set of 1260 observations. What is the estimated intercept for each group?

(b) Consider the hypothesis $H_0 : a_1 = 0$ versus $H_1 : a_1 \neq 0$. What does a rejection of this hypothesis mean?

(c) Using the results in Table 1, test the hypothesis $H_0 : a_1 = 0$ versus $H_1 : a_1 \neq 0$ at the 5% level.

(d) Table 2 gives the test statistics in the fitted values version of the test for heteroskedasticity (the White test). Is there evidence of heteroskedasticity in the model?

(e) Explain the null and how the test in part (d) is conducted.

Dependent Variable: LWAGE
Method: Least Squares

Sample: 1 1260
Included observations: 1260

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.869431	0.019724	94.77742	0.0000
FEMALE	-0.547765	0.033769	-16.22105	0.0000
BELAVG	-0.184294	0.057787	-3.189172	0.0015
FEMALE*BELAVG	0.033818	0.094293	0.358648	0.7199

R-squared	0.200553	Mean dependent var	1.658800
Adjusted R-squared	0.198644	S.D. dependent var	0.594508
S.E. of regression	0.532194	Akaike info criterion	1.579553
Sum squared resid	355.7379	Schwarz criterion	1.595867
Log likelihood	-991.1187	F-statistic	105.0287
Durbin-Watson stat	1.683915	Prob(F-statistic)	0.000000

Figure 1: Table 1: OLS Estimates

White Heteroskedasticity Test:

F-statistic	0.116428	Probability	0.950490
Obs*R-squared	0.350298	Probability	0.950307

Figure 2: Table 2: White Heteroskedasticity Test