Economics 471: Introductory Econometrics

Department of Economics, Finance and Legal Studies

University of Alabama

Fall 2017

Final Exam – Answers

1. (a) $\ln(y_i) = \ln\alpha + \beta \ln x_i + u_i$

   (b) $\ln\alpha$, β, and u_i, respectively

   (c) $\frac{1}{n}\sum_{i=1}^{n} \widehat{u}_i = \sum_{i=1}^{n}\left(\ln y_i - \widehat{\alpha} - \widehat{\beta}\ln x_i\right) = 0 \Rightarrow \widehat{\alpha} = \overline{ln\,(y)} - \widehat{\beta}\overline{ln\,(x)}$

   (d) $\frac{1}{n}\sum_{i=1}^{n} \widehat{u}_i x_i = \sum_{i=1}^{n}\left(\ln y_i - \widehat{\alpha} - \widehat{\beta}\ln x_i\right)x_i = 0 \Rightarrow \widehat{\beta} = \frac{\sum_{i=1}^{n}\left(\ln y_i - \overline{\ln y}\right)\left(\ln x_i - \overline{\ln x}\right)}{\sum_{i=1}^{n}\left(\ln x_i - \overline{\ln x}\right)^2}$

   (e) Given that we have logs on each side of the equation, the slope coefficient is interpreted as an elasticity.

2. (a) married men: $\alpha + \gamma$, married women: $\alpha + \delta$, single women: $\alpha + \lambda$ and single men: α

   (b) Single men are the base group as they are represented by the overall intercept.

   (c) This is a violation of Gauss-Markov assumption four: perfect collinearity. The model will not run as one variable is a perfect linear combination of one or more other variables.

   (d) The fact that they are all positive says that, holding x constant, each of the groups has a expected wage larger than single males. The ordering shows that, holding x constant, married men have higher expected wages than married females who have higher expected wages than single females.

   (e) Given the information we know from part (d), single women are the second highest expected wage group for a fixed level of x. Therefore we expect the coefficient on married men (γ) to remain positive, while the coefficients for single women (λ) and single men (ϕ) will be negative.

3. (a) $\sum_{i=1}^{n} \widehat{u}_i^2 = \sum_{i=1}^{n}\left(y_i - \widehat{\alpha} - \widehat{\beta}x_i\right)^2$. Setting $\partial \sum_{i=1}^{n} \widehat{u}_i^2 / \partial\widehat{\alpha} = 0$, and $\partial \sum_{i=1}^{n} \widehat{u}_i^2 / \partial\widehat{\beta} = 0$ and solving for $\widehat{\alpha}$ and $\widehat{\beta}$ we obtain $\widehat{\alpha} = \overline{y} - \widehat{\beta}\overline{x}$ and $\widehat{\beta} = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2}$ (note that the question only asks for the slope estimator). Heteroskedasticity does not affect the OLS estimates and thus we have the same exact estimator for $\widehat{\beta}$.

   (b) Assuming the first four Gauss-Markov assumptions hold, with homoskedasticity, OLS is BLUE: best linear unbiased estimator. Assuming the first four Gauss-Markov assumptions hold, with heteroskeasticity, OLS is unbiased, but inefficient.

   (c) $V\left(\widehat{\beta}\right) = V\left(\frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2}\right) = V\left(\frac{\sum_{i=1}^{n}(x_i - \overline{x})(\alpha + \beta x_i + u_i)}{\sum_{i=1}^{n}(x_i - \overline{x})^2}\right) = V\left(\frac{\sum_{i=1}^{n}(x_i - \overline{x})u_i}{\sum_{i=1}^{n}(x_i - \overline{x})^2}\right) = \frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \overline{x})^2}$.

(d) $V\left(\widehat{\beta}\right) = V\left(\frac{\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}{\sum_{i=1}^{n}(x_i-\overline{x})^2}\right) = V\left(\frac{\sum_{i=1}^{n}(x_i-\overline{x})(\alpha+\beta x_i+u_i)}{\sum_{i=1}^{n}(x_i-\overline{x})^2}\right) = V\left(\frac{\sum_{i=1}^{n}(x_i-\overline{x})u_i}{\sum_{i=1}^{n}(x_i-\overline{x})^2}\right) = \frac{\sum_{i=1}^{n}(x_i-\overline{x})^2\sigma_i^2}{\left[\sum_{i=1}^{n}(x_i-\overline{x})^2\right]^2}$.

(e) If we assume heteroskedasticity when it is actually homoskedastic, the estimator of the variance will be inefficient. If we assume homoskedasticity when it is actually heteroskedastic, the estimator of the variance will be biased.

4. (a) c(1): if you attend class 0% of the time, have a prior GPA of 0 and have an ACT score of 0, your predicted standardized test scores is -3.34, c(2): a one-percent increase in attendance results in a 0.005 standard deviation increase increase in the standardized final exam, c(3): a full point increase in your prior college grade point average results in a 0.40 standard deviation increase in the standardized final exam, c(4) a one point increase in your ACT score results in a 0.08 standard deviation increase in the standardized final exam

(b) Table 1: 0.005334, Table 2: $-0.006713 + 0.005586 * PRIGPA$

(c) Table 1: 0.402373, Table 2: $-1.628540 + 2 * 0.295905 * PRIGPA + 0.005586 * ATNDRTE$

(d) Table 1: 0.084257, Table 2: $-0.128039 + 2 * 0.004533 * ACT$

(e) $F = \frac{(SSR_R - SSR_u)/q}{SSR_u/(n-k-1)} = \frac{(530.9411-512.7624)/3}{512.7624/(680-3-1)} = \frac{\left(R_u^2 - R_R^2\right)/q}{\left(1-R_R^2\right)/(n-k-1)} = \frac{(0.228654-0.201308)/3}{(1-0.228654)/(680-6-1)} = 7.953135601 > F_{3,673,0.05} \approx F_{3,120,0.05} = 2.68$