Economics 471: Introductory Econometrics

Department of Economics, Finance and Legal Studies
University of Alabama
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Final Exam — Key

The exam consists of four questions on five pages. Each question is of equal value.

1. Consider a regression model without an intercept: $y_i = \beta x_i + u_i$, $i = 1, 2, \ldots, n$. Given this information, answer the following:

    (a) Derive the method of moments estimator of β.

    (b) Derive the variance of the estimator achieved in part (a).

    (c) For this model, give the exact form of SSE, SSR and SST.

    (d) Suppose that there were two groups in the population (group 0 and group 1) and we wished to account for this in our model by using a dummy variable (D) via $y_i = \beta x_i + \delta D_i + u_i$. On a single figure, draw a reasonable regression line for each group.

    (e) For the model in part (d), how would you test the null hypothesis that there is no difference between the two groups (state the null, test statistic, distribution of the test statistic, and decision rule)?

(a) $E(ux) = 0 \Rightarrow \frac{1}{n}\sum_{i=1}^{n} \hat{u}_i x_i = 0 \Rightarrow$

$\frac{1}{n}\sum_{i=1}^{n}(y_i - x_i\hat{\beta})x_i = 0 \Rightarrow \frac{1}{n}\sum_{i=1}^{n} y_i x_i - \frac{1}{n}\sum_{i=1}^{n}\hat{\beta}x_i^2 = 0$

$\Rightarrow \hat{\beta} = \dfrac{\sum_{i=1}^{n} y_i x_i}{\sum_{i=1}^{n} x_i^2}$

(b) $\hat{\beta} = \dfrac{\sum_{i=1}^{n} y_i x_i}{\sum_{i=1}^{n} x_i^2} = \dfrac{\sum(\beta x_i + u_i)x_i}{\sum_{i=1}^{n} x_i^2} = \beta + \dfrac{\sum_{i=1}^{n} u_i x_i}{\sum_{i=1}^{n} x_i^2}$

$V(\hat{\beta}|x) = V\left(\beta + \dfrac{\sum_{i=1}^{n} u_i x_i}{\sum_{i=1}^{n} x_i^2} \Big| x\right) = 0 + \dfrac{1}{\left(\sum_{i=1}^{n} x_i^2\right)^2} V\left(\sum_{i=1}^{n} u_i x_i | x\right)$

$\overset{\text{iid}}{=} \dfrac{1}{\left(\sum_{i=1}^{n} x_i^2\right)^2} \sum_{i=1}^{n} V(u_i x_i | x) = \dfrac{1}{\left(\sum_{i=1}^{n} x_i^2\right)^2} \sum_{i=1}^{n} x_i^2 V(u_i | x)$

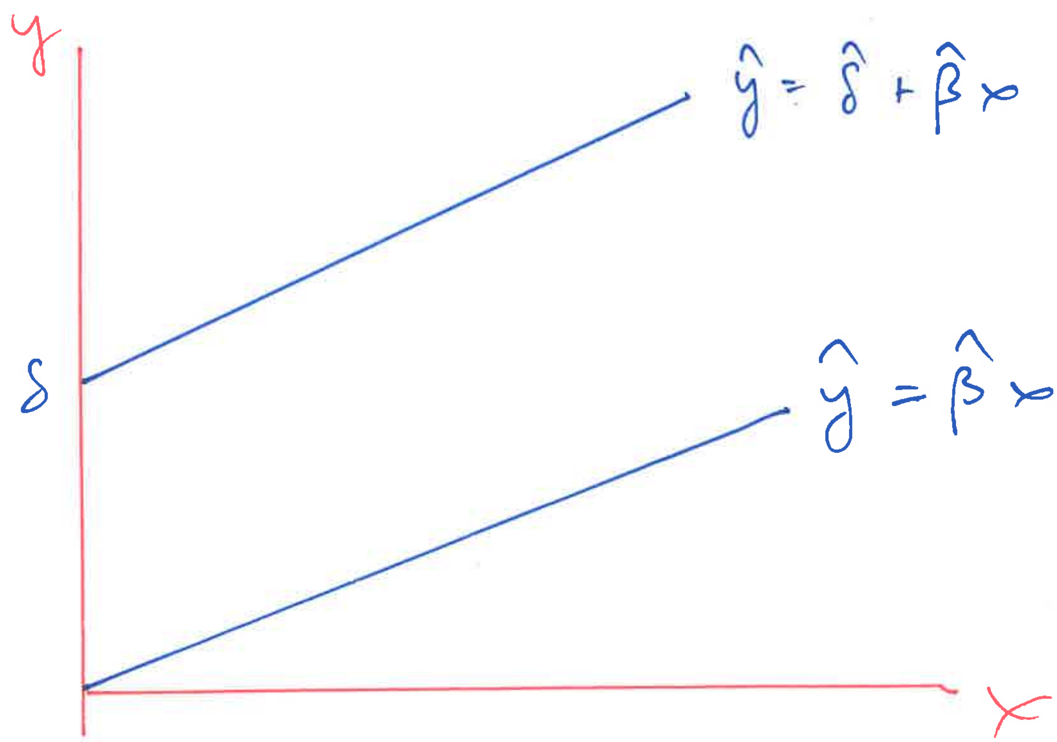
$= \dfrac{1}{\left(\sum_{i=1}^{n} x_i^2\right)^2} \sum_{i=1}^{n} \sigma^2 x_i^2 = \dfrac{\sigma^2}{\sum_{i=1}^{n} x_i^2}$

(c) $SSE = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2 = \sum_{i=1}^{n} (x_i \hat{\beta} - \bar{y})^2$

$SSR = \sum_{i=1}^{n} \hat{u}_i^2 = \sum_{i=1}^{n} (y_i - \hat{\beta} x_i)^2$

$SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$

(d)



(e) $H_0: \delta = 0$

$H_a: \delta \neq 0$

$$t = \frac{\hat{\delta} - 0}{se(\hat{\delta})} \sim t_{n-2}$$

reject H_0 if $|t| > t_{\alpha,(n-2)}$ (or 2)

2. Suppose your model is $y_i = \alpha + \beta x_i + \gamma x_i^2 + u_i$, but the true data generating process is $y_i = \alpha + \beta x_i + \varepsilon_i$. With this information, answer the following:

(a) What Gauss-Markov assumption(s) are violated in the model?

(b) What are the consequences of the violation(s) from part (a)?

(c) Without deriving the result, what is the expected value of the estimator of β from the model?

(d) Without deriving the result, what is the expected value of the estimator of γ from the model?

(e) For this model, how would you test the null hypothesis that y is a linear function of x (state the null, test statistic, distribution of the test statistic, and decision rule)?

(a) assumption 1: the model is correctly specified

(b) OLS estimators are not BLUE (inefficient)

(c) $E(\hat{\beta}|x) = \beta$

(d) $E(\hat{\gamma}|x) = \gamma = 0$

(e) $H_0: \gamma = 0$

$H_1: \gamma \neq 0$

$t = \dfrac{\hat{\gamma} - 0}{se(\hat{\gamma})} \sim t_{n-3}$

reject H_0 if $|t| > t_{n-3}$ (or z)

3. Consider a random sample of data $\{y_i\}_{i=1}^{n}$ that comes from two groups (1 and 0) of size n_1 and n_0, respectively (where $n = n_1 + n_0$). Suppose we propose the model $y_i = \alpha + \delta D_i + u_i$, where $E(u_i|D_i) = 0$ and the error variances are $V(u_i|D_i = 1) = \sigma_1^2$ and $V(u_i|D_i = 1) = \sigma_0^2$ for groups 1 and 0, respectively (where $\sigma_1^2 > \sigma_0^2$). With this information, answer the following:

(a) Derive the method of moments estimator for α.

(b) What is the expected change in y when switching from group 0 to group 1 (i.e., $E(y_i|D_i = 1) - E(y_i|D_i = 0)$)?

(c) Propose an estimator for the overall error variance, σ^2.

(d) Propose an estimator for error variance for group 1, σ_1^2. Propose an estimator for the error variance for group 0, σ_0^2.

(e) On a single figure, draw a reasonable scatter plot for each group (use different colors or symbols for each group).

(a) $E(u) = 0 \Rightarrow \frac{1}{n}\sum_{i=1} \hat{u}_i = 0 \Rightarrow \frac{1}{n}\sum_{i=1} (y_i - \hat{\alpha} - \hat{\delta} D_i) = 0$

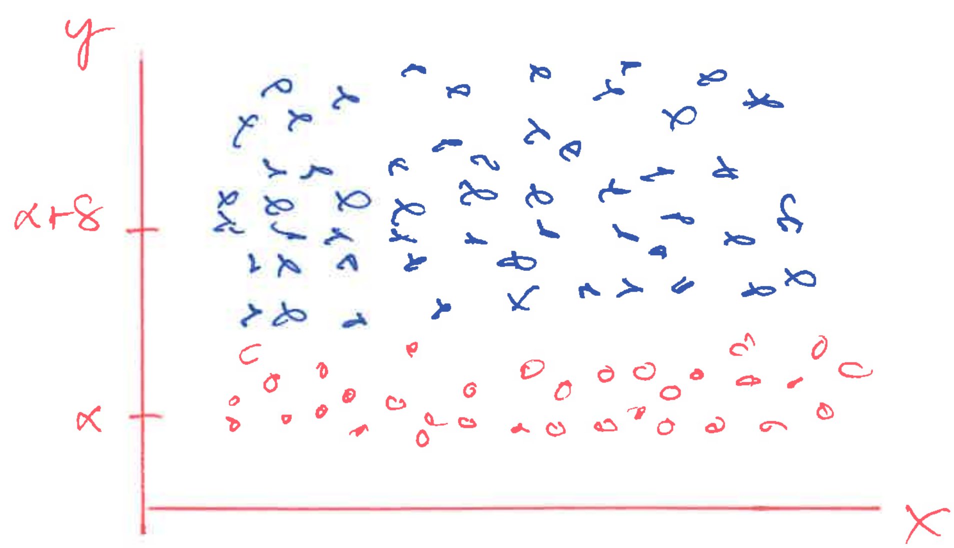
$\Rightarrow \bar{y} - \hat{\alpha} - \hat{\delta}\bar{D} = 0 \Rightarrow \hat{\alpha} = \bar{y} - \hat{\delta}\bar{D}$

(b) $E(y_i|D_i = 1) - E(y_i|D_i = 0) = (\alpha + \delta) - (\alpha) = \delta$

(c) $\hat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1} \hat{u}_i^2$

(d) $\hat{\sigma}_1^2 = \frac{1}{n_1 - 2}\sum_{i=1}^{n_1} \hat{u}_i^2 \qquad \hat{\sigma}_0^2 = \frac{1}{n_0 - 2}\sum_{i=1}^{n_0} \hat{u}_i^2$

(e)



$\hat{\sigma}_1^2 > \hat{\sigma}_0^2$

4. List the six Gauss-Markov Assumptions that we discussed in class. Then, consider the four pieces of gretl output below. For each piece of output, list the assumption(s) that are being checked.

(1) the model is correctly specified

$y = \alpha + \beta x + u$

(2) errors are random

$E(u) = 0$

(3) errors are uncorrelated w/ regressors

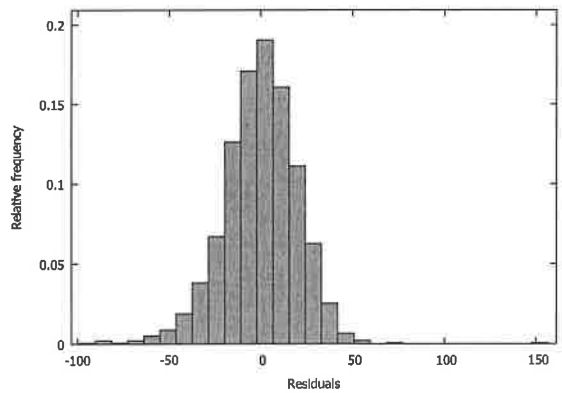
$E(u|x) = 0$

(4) no perfect collinearity

(5) homoskedasticity

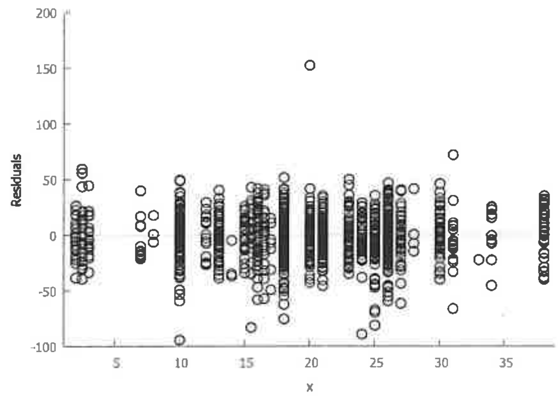
$V(u|x) = \sigma^2$

(6) $u|x \overset{iid}{\sim} N(0, \sigma^2)$



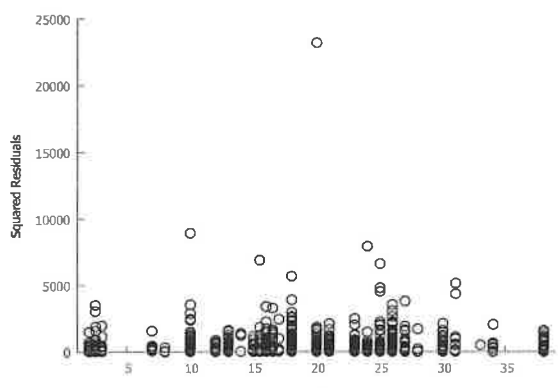
Relative frequency
Residuals

(2), (6)



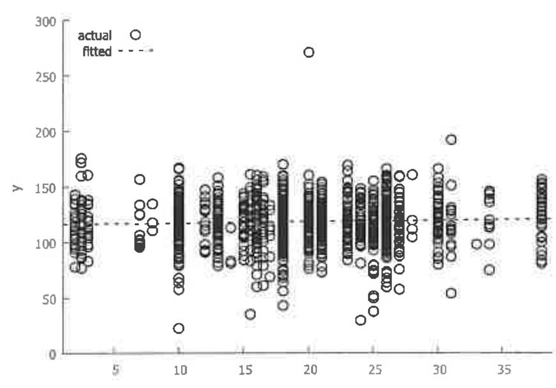
Residuals
x

(2), (3)



Squared Residuals
x

(5), (6)



actual
fitted
y
x

(1)