Economics 413: Economic Forecast & Analysis

Department of Economics, Finance and Legal Studies

University of Alabama

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Midterm II – Answers

1. (a) $y_t = c + \phi y_{t-1} + \beta_1 x_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$

   (b) The number of spikes in the PACF and the number of spikes in the ACF, respectively.

   (c) The number of spikes in the CCF (cross-correlation function).

   (d) $y_{t+h} = c + \phi y_{t+h-1} + \beta_1 x_{t+h-1} + \varepsilon_{t+h} + \theta\varepsilon_{t+h-1}$

   (e) $\widehat{y}_{t+h|t} = E\left(y_{t+h}|\Omega_t\right) = E\left(c + \phi y_{t+h-1} + +\beta_1 x_{t+h-1} + \varepsilon_{t+h} + \theta\varepsilon_{t+h-1}|\Omega_t\right) = \widehat{c} + \widehat{\phi}\widehat{y}_{t+h-1|t} + \widehat{\beta}_1\widehat{x}_{t+h-1|t}$. Given that we have set $h = 1$, $\widehat{y}_{t+1|t} = \widehat{c} + \widehat{\phi}\widehat{y}_{t+1-1|t} + \widehat{\beta}_1\widehat{x}_{t+1-1|t} = \widehat{c} + \widehat{\phi}y_t + \widehat{\beta}_1 x_t$, where we dropped the hats on y_t and x_t because they are known at time period t.

2. (a) $\widehat{Y}_{t+1|t} = E\left(Y_{t+1}|\Omega_t\right) = E\left(\mu + \varepsilon_{t+1} + \theta_3\varepsilon_{t-2}\right) = \mu + \theta_3\varepsilon_{t-2}$, $\widehat{Y}_{t+2|t} = E\left(Y_{t+2}|\Omega_t\right) = E\left(\mu + \varepsilon_{t+2} + \theta_3\varepsilon_{t-1}\right) = \mu + \theta_3\varepsilon_{t-1}$, $\widehat{Y}_{t+3|t} = E\left(Y_{t+3}|\Omega_t\right) = E\left(\mu + \varepsilon_{t+3} + \theta_3\varepsilon_t\right) = \mu + \theta_3\varepsilon_t$, $\widehat{Y}_{t+4|t} = E\left(Y_{t+4}|\Omega_t\right) = E\left(\mu + \varepsilon_{t+4} + \theta_3\varepsilon_{t+1}\right) = \mu$, $\widehat{Y}_{t+h|t} = E\left(Y_{t+h}|\Omega_t\right) = E\left(\mu + \varepsilon_{t+h} + \theta_3\varepsilon_{t+h-3}\right) = \mu \ \forall h > 3$

   (b) $e_{t+1} = Y_{t+1} - \widehat{Y}_{t+1|t} = \mu + \varepsilon_{t+1} + \theta_3\varepsilon_{t-2} - \left(\mu + \theta_3\varepsilon_{t-2}\right) = \varepsilon_{t+1}$, $e_{t+2} = Y_{t+2} - \widehat{Y}_{t+2|t} = \mu + \varepsilon_{t+2} + \theta_3\varepsilon_{t-1} - \left(\mu + \theta_3\varepsilon_{t-1}\right) = \varepsilon_{t+2}$, $e_{t+3} = Y_{t+3} - \widehat{Y}_{t+3|t} = \mu + \varepsilon_{t+3} + \theta_3\varepsilon_t - \left(\mu + \theta_3\varepsilon_t\right) = \varepsilon_{t+3}$, $e_{t+4} = Y_{t+4} - \widehat{Y}_{t+4|t} = \mu + \varepsilon_{t+4} + \theta_3\varepsilon_{t+1} - \mu = \varepsilon_{t+4} + \theta_3\varepsilon_{t+1}$, $e_{t+h} = Y_{t+h} - \widehat{Y}_{t+h|t} = \mu + \varepsilon_{t+h} + \theta_3\varepsilon_{t+h-3} - \mu = \varepsilon_{t+h} + \theta_3\varepsilon_{t+h-3} \ \forall h > 3$

   (c) $V\left(e_{t+1}\right) = V\left(\varepsilon_{t+1}\right) = \sigma^2$, $V\left(e_{t+2}\right) = V\left(\varepsilon_{t+2}\right) = \sigma^2$, $V\left(e_{t+3}\right) = V\left(\varepsilon_{t+3}\right) = \sigma^2$, $V\left(e_{t+4}\right) = V\left(\varepsilon_{t+4} + \theta_3\varepsilon_{t+1}\right) = \sigma^2\left(1 + \theta_3^2\right)$, $V\left(e_{t+h}\right) = V\left(\varepsilon_{t+h} + \theta_3\varepsilon_{t+h-3}\right) = \sigma^2\left(1 + \theta_3^2\right) \ \forall h > 3$

   (d) $\left[\widehat{Y}_{t+h|t} \pm 2\sigma\right]$ for $h = 1, 2, 3$ and $\left[\widehat{Y}_{t+h|t} \pm 2\sigma\sqrt{(1 + \theta_3^2)}\right] \ \forall h > 3$

   (e) The figure should be an MA model with mean of μ, point forecast estimates equal to that in part (a) and interval estimates equal to that in part (d).

2. (a) MA(4): all MA models are stationary, AR(3): $|0.281| + |0.345| + |0.177| = 0.803 < 1$, AR(4): $|0.238| + |0.261| + |0.111| + |0.284| = 0.894 < 1$, AR(5): $|0.282| + |0.270| + |0.135| + |0.307| + |-0.130| = 1.124 > 1$, but $0.282 + 0.270 + 0.135 + 0.307 + (-0.130) = 0.864 < 1$ so the necessary condition holds (hence, ambigious), ARMA(2,2): $|0.134| + |0.777| = 0.911 < 1$, ARMA(2,4): $|0.297| + |0.580| = 0.877 < 1$

   (b) The residual variance is smaller for the AR(5) model and the adjusted r-squared is largest for the AR(5) model. The AIC and SIC are smallest for the AR(4) model.

   (c) AR(5): $\widehat{y}_t = 0.282y_{t-1} + 0.270y_{t-2} + 0.135y_{t-3} + 0.307y_{t-4} - 0.130y_{t-5}$, AR(4):$\widehat{y}_t = 0.238y_{t-1} + 0.261y_{t-2} + 0.111y_{t-3} + 0.284y_{t-4}$

(d) AR(5): yes, the p-values for the Q-statistics are 0.4 and 0.2 which are both greater than 0.05, AR(4): yes, the p-values for the Q-statistics are 0.1 and 0.2 which are both greater than 0.05

(e) AR(5): $\widehat{y}_{t+1|t} = 0.282 y_t + 0.270 y_{t-1} + 0.135 y_{t-2} + 0.307 y_{t-3} - 0.130 y_{t-4}$, AR(4): $\widehat{y}_{t+1|t} = 0.238 y_t + 0.261 y_{t-1} + 0.111 y_{t-2} + 0.284 y_{t-3}$