Economics 413: Economic Forecast & Analysis

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Midterm II – Answers

1. (a) ARMA(0,0)
   (b) Mean, variance and covariance do not depend upon time
   (c) $E(Y_t) = E(\blacklozenge + \varepsilon_t) = \blacklozenge$
   (d) $V(Y_t) = E\left[(Y_t - \mu)^2\right] = E\left[(Y_t - \blacklozenge)^2\right] = E\left(\varepsilon_t^2\right) = \sigma^2$
   (e) $COV(Y_t, Y_{t-j}) = E[(Y_t - \mu)(Y_{t-j} - \mu)] = E[(Y_t - \blacklozenge)(Y_{t-j} - \blacklozenge)] = E(\varepsilon_t \varepsilon_{t-j}) = 0 \forall j \neq 0$

2. (a) $\ln L(\Theta) = -\frac{T}{2}\ln 2\pi - \frac{T}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum_{t=1}^{T}(y_t - c - \phi y_{t-1} - \theta\varepsilon_{t-1})^2$
   (b) In OLS we want to minimize $SSR = \sum_{t=1}^{T}\varepsilon_t^2$. Here $\ln L(\Theta)$ is a function of $-SSR$ and hence we want to maximize $-\sum_{t=1}^{T}(y_t - c - \phi y_{t-1} - \theta\varepsilon_{t-1})^2$.
   (c) $\ln L(\Theta) = -\frac{T}{2}\ln 2\pi - \frac{T}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum_{t=1}^{T}(y_t - c - \phi y_{t-1})^2$
   (d) $\sum_{t=1}^{T}\varepsilon_t^2 = \sum_{t=1}^{T}(y_t - \phi y_{t-1})^2$
   (e) $\frac{\partial\sum_{t=1}^{T}\varepsilon_t^2}{\partial\phi} = -2\sum_{t=1}^{T}(y_t - \phi y_{t-1})y_{t-1} = 0 \Longrightarrow \sum_{t=1}^{T}\left(y_t y_{t-1} - \phi y_{t-1}^2\right) = 0 \Longrightarrow \widehat{\phi} = \frac{\sum_{t=1}^{T} y_t y_{t-1}}{\sum_{t=1}^{T} y_{t-1}^2}$

3. $Y_{t+h} = \mu + \beta X_{t+h} + \varepsilon_{t+h}$
   (a) $\widehat{Y}_{t+h|t} = E(Y_{t+h}|\Omega_t) = E(\mu + \beta X_{t+h} + \varepsilon_{t+h}|\Omega_t) = \mu \ \forall h$
   (b) $e_{t+h} = Y_{t+h} - \widehat{Y}_{t+h|t} = \mu + \beta X_{t+h} + \varepsilon_{t+h} - \mu = \beta X_{t+h} + \varepsilon_{t+h} \ \forall h$
   (c) $V(e_{t+h}) = V(\beta X_{t+h} + \varepsilon_{t+h}) = \beta^2\sigma_X^2 + \sigma_\varepsilon^2 \ \forall h$
   (d) $P\left[\widehat{Y}_{t+h|t} - 2\sqrt{\beta^2\sigma_X^2 + \sigma_\varepsilon^2} < Y_{t+h} < \widehat{Y}_{t+h|t} + 2\sqrt{\beta^2\sigma_X^2 + \sigma_\varepsilon^2}\right] =$
   $P\left[\mu - 2\sqrt{\beta^2\sigma_X^2 + \sigma_\varepsilon^2} < Y_{t+h} < \mu + 2\sqrt{\beta^2\sigma_X^2 + \sigma_\varepsilon^2}\right] \approx 0.95 \ \forall h$
   (e) A plot of Y versus t will show straight lines for each. Part (a) is a straight line equal to μ and the upper and lower bounds will be also be straight lines equal to μ plus and minus $2\sqrt{\beta^2\sigma_X^2 + \sigma_\varepsilon^2}$.

4. (a) $H_0 : \theta_3 = \theta_4 = \theta_5 = \theta_6 = 0$

   (b) The second table adds additional lags of the error term (specifically the estimated residuals) on the right hand side. If those coefficients are significant, then it implies that the first model did not remove all of the correlation in the error and hence the residuals are not white noise.

   (c) There is no autocorrelation in the residuals. Correctly specified.

   (d) There is autocorrelation in the residuals. Incorrect specification.

   (e) If we fail to reject then we move onto forecasting. If we reject the null we must find a new tenative model.