Economics 413: Economic Forecast & Analysis

Department of Economics, Finance and Legal Studies

University of Alabama

Fall 2012

Midterm II – Answers

1. Box-Jenkins methodology (see Handout #1 for the flow chart)

   (a) Difference the series to achieve stationarity: We first need the series to be stationary. This can be done by either differencing a series (usually works with economic data) or adding time trends and/or seasonal dummies.

   (b) Identify model to be tentatively entertained: Once we have a stationary time series, we examine the correlogram to decide on the appropriate orders of the AR and MA components. Based on these, we can arrive at a tentative ARMA model.

   (c) Estimate the parameters of the tentative model: ARMA models are generally estimated by MLE.

   (d) Diagnostic checking: We need to check that there is no serial correlation in the residuals. This can be done several ways (correlogram of residuals, Q-statistics, Box-Ljung statistics or LM tests).

   (e) Use the model for forecasting: Once we have our model, we can use it to perform forecasts. Estimate the model, get the value for $\widehat{y}$ at time period t and then put that back into the model to get the predicted value for $\widehat{y}$ at time period $t+1$ and so on.

See figure

2. (a) $\widehat{Y}_{t+1|t} = E\left(Y_{t+1}|\Omega_t\right) = E\left(\varepsilon_{t+1} + \theta_2\varepsilon_{t-1}|\Omega_t\right) = \theta_2\varepsilon_{t-1}$, $\widehat{Y}_{t+2|t} = E\left(Y_{t+2}|\Omega_t\right) = E\left(\varepsilon_{t+2} + \theta_2\varepsilon_t|\Omega_t\right) = \theta_2\varepsilon_t$, $\widehat{Y}_{t+h|t} = E\left(Y_{t+h}|\Omega_t\right) = E\left(\varepsilon_{t+h} + \theta_2\varepsilon_{t+h-2}|\Omega_t\right) = 0 \forall h > 2$

   (b) $e_{t+1} = Y_{t+1} - \widehat{Y}_{t+1|t} = \varepsilon_{t+1}$, $e_{t+2} = Y_{t+2} - \widehat{Y}_{t+2|t} = \varepsilon_{t+2}$, $e_{t+h} = Y_{t+h} - \widehat{Y}_{t+h|t} = \varepsilon_{t+h} + \theta_2\varepsilon_{t+h-2} \forall h > 2$

   (c) $V\left(e_{t+1}\right) = V\left(\varepsilon_{t+1}\right) = \sigma^2$, $V\left(e_{t+2}\right) = V\left(\varepsilon_{t+2}\right) = \sigma^2$, $V\left(e_{t+h}\right) = V\left(\varepsilon_{t+h}\right) = \sigma^2\left(1+\theta_2^2\right) \forall h > 2$

   (d) $\widehat{Y}_{t+1|t} \pm 1.96\sigma$, $\widehat{Y}_{t+2|t} \pm 1.96\sigma$, $\widehat{Y}_{t+h|t} \pm 1.96\sqrt{\sigma^2\left(1+\theta_2^2\right)} \forall h > 2$

   (e) First two forecasts are different from zero, but zero afterwards. The confidence bound is the same width for $h = 1$ and 2 and increases at 3, but it constant at that point (it does *not* grow in size as compared to AR models).

See figure

3. (a) $\widehat{y}_{2025|2010} = \widehat{\mu}$. This is simply a dot above 2025 at the mean of the data (use best guess here).

   (b) $e_{2025} = y_{2025} - \widehat{y}_{2025|2010} = \mu + \varepsilon_{2025} + \widehat{\theta}\varepsilon_{2024} - \widehat{\mu} \approx \varepsilon_{2025} + \widehat{\theta}\varepsilon_{2024}$. $V\left(e_{2025}\right) = E\left[\left(\varepsilon_{2025} + \theta\varepsilon_{2024}\right)^2\right] = \left(1+\theta^2\right)\sigma^2$, which we approximate with $\left(1+\widehat{\theta}^2\right)\widehat{\sigma}^2$. Assuming normality, our interval forecast becomes $\left[\widehat{\mu} \pm 1.96\sqrt{\left(1+\widehat{\theta}^2\right)\widehat{\sigma}^2}\right]$. On the figure it is centered over the point forecast and extends above and below.

   (c) This question only asks you to draw the density forecast. It will be centered at the point estimate and will overlap (and extend beyond, by 2.5% on each side) the interval forecast.

Handout #1

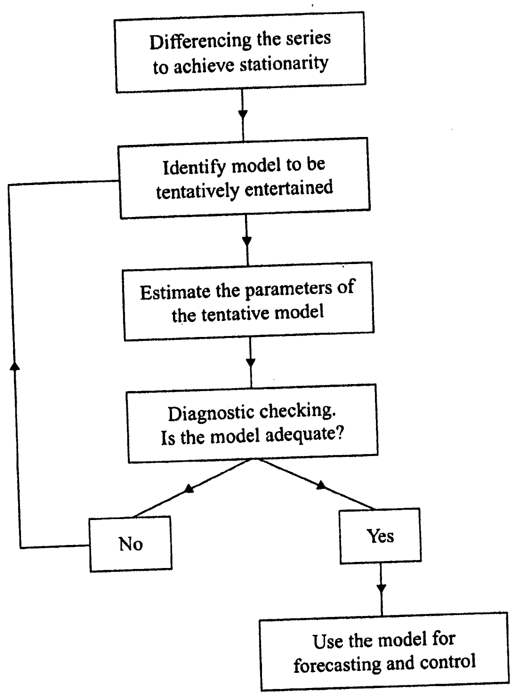
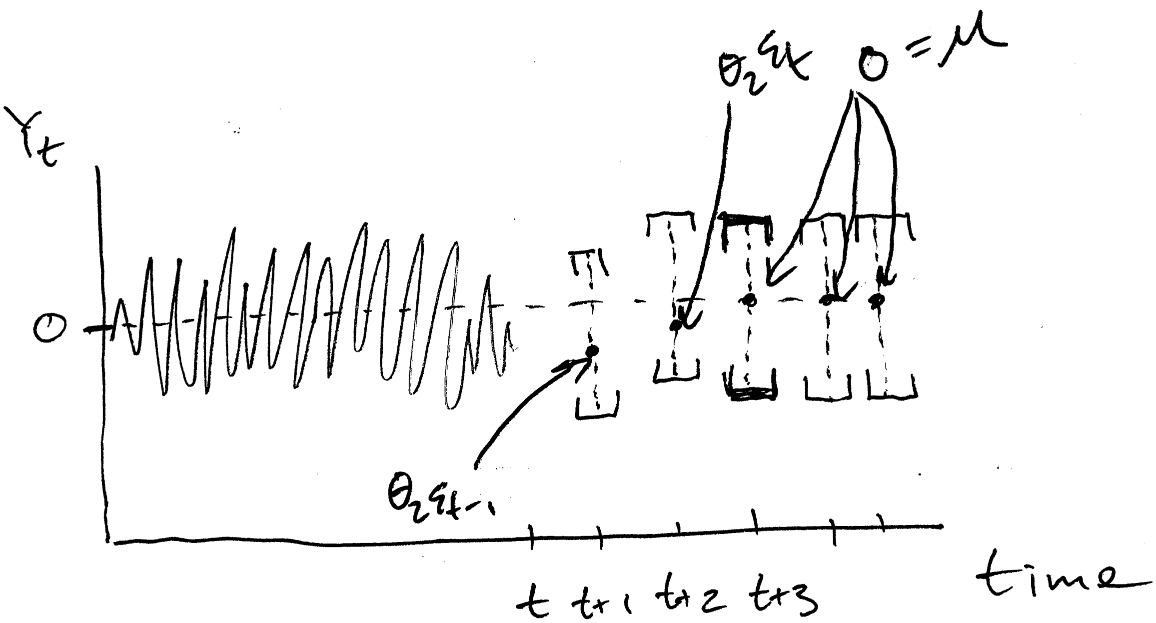


Figure 12.3 The Box–Jenkins methodology for ARIMA models.

(2e)



confidence band values are on sheet, must be same for 1 & 2, & for 3,4, ..., but 1 & 2 smaller than 3,4, ...

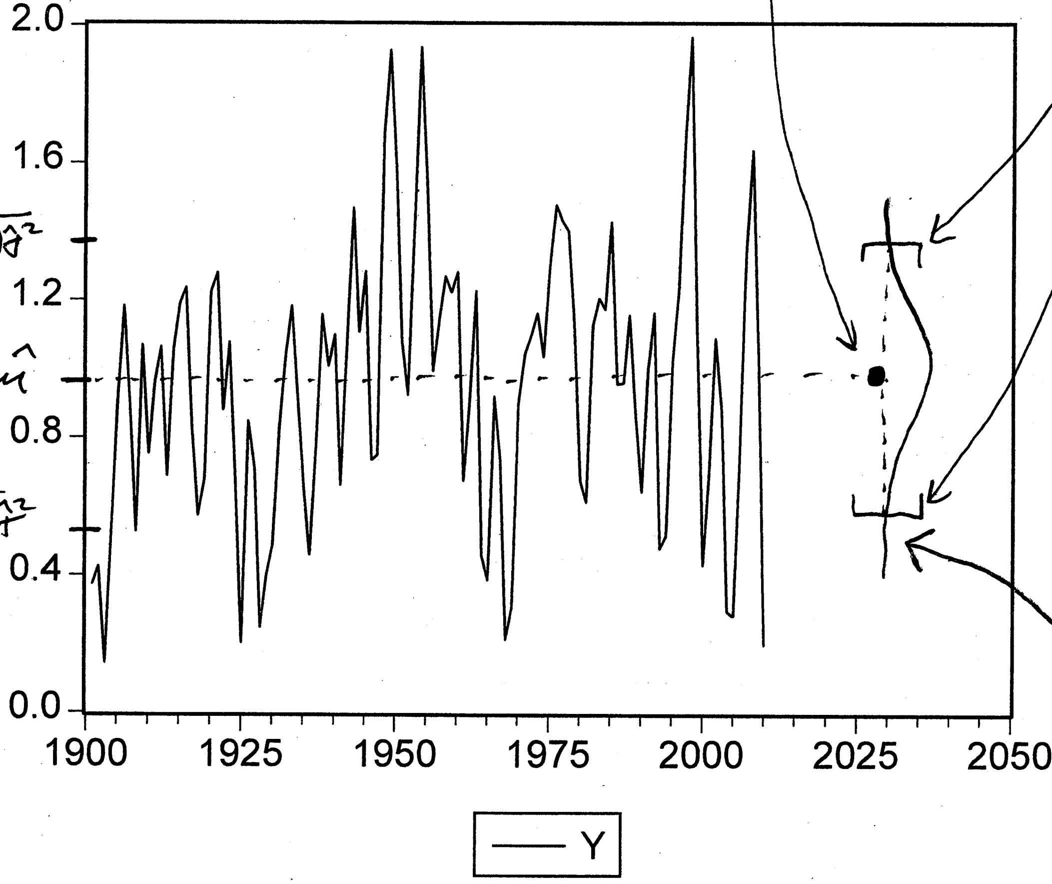
point forecast $\hat{y}_{2025|2010} = \hat{\mu}$

interval forecast

$\hat{\mu} + 1.96\sqrt{(1+\theta^2)\sigma^2}$

$\hat{\mu}$

$\hat{\mu} - 1.96\sqrt{(1+\theta^2)\sigma^2}$



density forecast

must extend beyond the CI, the CI is only 95% of the area