Economics 413: Economic Forecast and Analysis

Department of Economics, Finance and Legal Studies

University of Alabama

Fall 2023

Midterm I – Key

The exam consists of three questions on three pages. Each question is of equal value.

1. Consider the following model: $Y_t = 1 + \varepsilon_t + 2\varepsilon_{t-1}$, where $\varepsilon_t \sim WN$. With this information, answer the following:

   (a) Derive the expected value of the series.

   (b) Derive the variance of the series.

   (c) Derive the autocovariance of the series for all lags $j = 1, 2, \ldots$.

   (d) Derive the autocorrelation of the series for all lags $j = 1, 2, \ldots$.

   (e) Is this series stationary? Is this series invertible? How do you know?

(a) $E(Y_t) = E(1 + \varepsilon_t + 2\varepsilon_{t-1}) = 1$

(b) $\gamma_0 = E[(Y_t - \mu)^2] = E[(\varepsilon_t + 2\varepsilon_{t-1})^2]$

$= E(\varepsilon_t^2 + 4\varepsilon_{t-1}^2 + 4\varepsilon_t\varepsilon_{t-1}) = \sigma^2 + 4\sigma^2$

(c) $\gamma_j = E[(Y_t - \mu)(Y_{t-j} - \mu)] = E[(\varepsilon_t + 2\varepsilon_{t-1})(\varepsilon_{t-j} + 2\varepsilon_{t-j-1})]$

$= \begin{cases} 2\sigma^2 & \text{for } j = 1 \\ 0 & \text{for } j > 1 \end{cases}$

(d) $\rho_j = \dfrac{\gamma_j}{\gamma_0} = \begin{cases} \dfrac{2}{(1+4)} & \text{for } j = 1 \\ 0 & \text{for } j > 1 \end{cases}$

(e) Yes, No (All MA processes are stationary)
($\theta = 2 > 1 \Rightarrow$ not invertible)

2. In the figure below, we have four realizations of a stochastic process. With this information, answer the following:

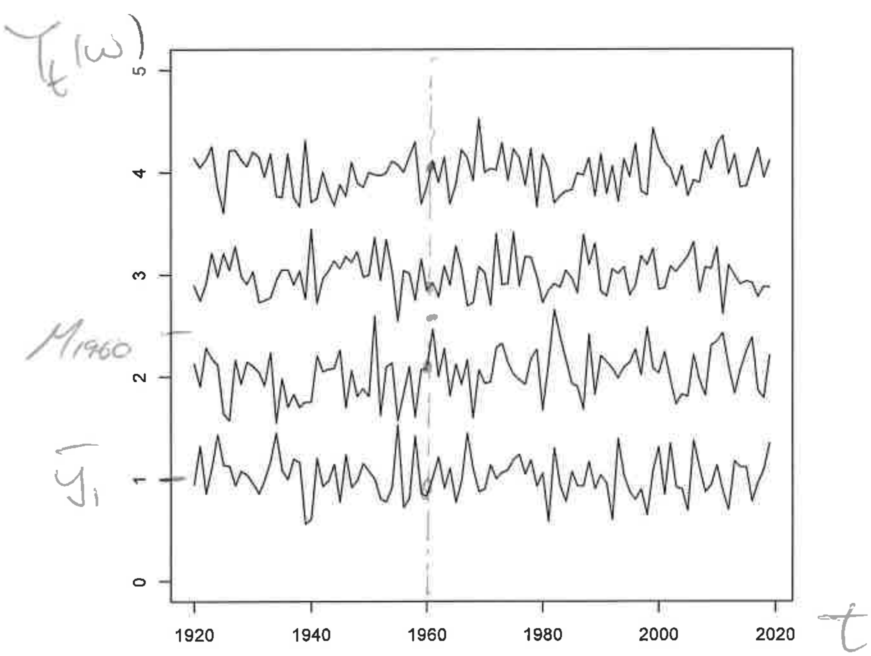
   (a) Label the axes. $Y_t(\omega)$ & t

   (b) Write the formula to calculate the ensemble average for the year 1960 (i.e., the expectation of the stochastic process in the year 1960). Give a reasonable estimate of this average.

   (c) Write the formula to calculate the time series average for *a* realization of the stochastic process. For one of the realizations, give a reasonable estimate of this average.

   (d) Write the formula to calculate the variance of the stochastic process for the year 1960 (i.e., the variance of the stochastic process in the year 1960). Give a reasonable estimate of this variance?

   (e) Write the formula to calculate the variance of the time series for *a* realization of the stochastic variance. For one of the realizations, give a reasonable estimate of this variance.



(b) $\mu_t \equiv E(Y_t) = \frac{1}{4} \sum_{i=1}^{4} Y_t(i)$

$\mu_t \approx 2.5$

(c) $\bar{y} = \frac{1}{T} \sum_{t=1}^{T} y_t$

for $i=1$ $\bar{y}_1 \approx 1$

(d) $\gamma_{0t} \equiv V(Y_t) = \frac{1}{4} \sum_{i=1}^{4} [Y_t(i) - \mu_t]^2$

$\gamma_{0\,1960} = V(Y_{1960}) \approx \frac{1}{4} [(Y_{1960}(1) - 2.5)^2$

$+ (Y_{1960}(2) - 2.5)^2 + (Y_{1960}(3) - 2.5)^2$

$+ (Y_{1960}(4) - 2.5)^2]$

$\approx \frac{1}{4} [(1-2.5)^2 + (2-2.5)^2 + (3-2.5)^2 + (4-2.5)^2]$

≈ 2.5

(e) $\hat{\sigma}_y^2 = \frac{1}{T-1} \sum_{t=1}^{T} (y_t - \bar{y})^2$

for $i=1$ $\hat{\sigma}_{y_1}^2 \approx 0.5$

3. Consider the gretl output listed below. For quarterly data of the returns of Bank of New York Mellon Bank stock (*melret*), the sample ACF and PACF are given. With this information, answer the following:

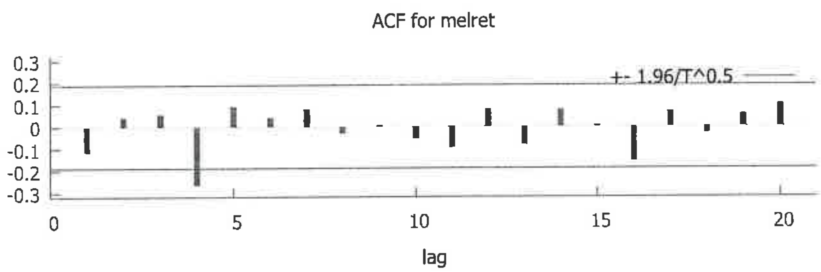
(a) What does a given spike in the sample ACF measure? What does a given spike in the sample PACF measure?

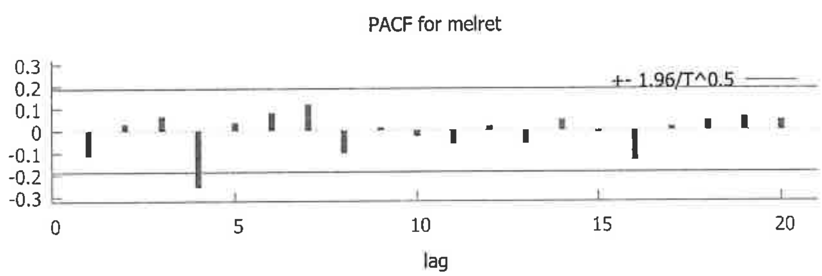
(b) Using this sample ACF and PACF, what is a reasonable model to be entertained?

(c) What is the expected value of the model you listed in part (b)?

(d) Under what conditions is the model listed in part (b) stationary?

(e) For the model you listed in part (b), draw the theoretical ACF and PACF.





(a) $\hat{\rho}_j = \widehat{CORR}(y_t, y_{t-j})$

$\hat{\phi}_{jj} = \widehat{CORR}(y_t, y_{t-j} \mid y_{t-1}, y_{t-2}, \ldots, y_{t-j+1})$

(b) $y_t = c + \phi_4 y_{t-4} + \varepsilon_t, \; \varepsilon_t \sim WN$

(other reasonable models are acceptable)

(c) assuming stationarity

$$E(Y_t) = E(c + \phi_1 Y_{t-1} + \varepsilon_t)$$

$$\mu = c + \phi_1 \mu$$

$$\mu = \frac{c}{1-\phi_1}$$

(d) $|\phi_1| < 1 \Rightarrow$ stationarity

(e)