Economics 413: Economic Forecast & Analysis

Department of Economics, Finance and Legal Studies

University of Alabama

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Midterm I – Answers

1. (a) ARMA(0,0)

   (b) Mean, variance and covariance do not depend upon time

   (c) $E(Y_t) = E(c + \varepsilon_t) = c$

   (d) $V(Y_t) = E\left[(Y_t - \mu)^2\right] = E\left[(Y_t - c)^2\right] = E\left(\varepsilon_t^2\right) = \sigma^2$

   (e) $COV(Y_t, Y_{t-j}) = E[(Y_t - \mu)(Y_{t-j} - \mu)] = E[(Y_t - c)(Y_{t-j} - c)] = E(\varepsilon_t \varepsilon_{t-j}) = 0 \forall j \neq 0$

2. (a) $E(Y_t) = E(c + \phi_2 Y_{t-2} + \varepsilon_t) \Rightarrow \mu = c + \phi_2 \mu \Rightarrow \mu = c/(1 - \phi_2)$

   (b) $V(Y_t) = E\left[(Y_t - \mu)^2\right] = E\left\{[\phi_2(Y_{t-2} - \mu) + \varepsilon_t]^2\right\} = \phi_2^2 \gamma_0 + \sigma^2 \Rightarrow \gamma_0 = \sigma^2/\left(1 - \phi_2^2\right)$

   (c) $COV(Y_t, Y_{t-1}) = E[(Y_t - \mu)(Y_{t-1} - \mu)] = E\{[\phi_2(Y_{t-2} - \mu) + \varepsilon_t](Y_{t-1} - \mu)\} = \phi_2 \gamma_1 \Rightarrow \gamma_1 = 0$
   $COV(Y_t, Y_{t-2}) = E[(Y_t - \mu)(Y_{t-2} - \mu)] = E\{[\phi_2(Y_{t-2} - \mu) + \varepsilon_t](Y_{t-2} - \mu)\} = \phi_2 \gamma_0$ $COV(Y_t, Y_{t-j}) = E[(Y_t - \mu)(Y_{t-j} - \mu)] = E\{[\phi_2(Y_{t-2} - \mu) + \varepsilon_t](Y_{t-j} - \mu)\} = \phi_2 \gamma_{j-2} \forall j > 1$

   (d) $\rho_j = \gamma_j/\gamma_0$, $\rho_0 = \gamma_0/\gamma_0 = 1$, $\rho_1 = \gamma_1/\gamma_0 = 0$, $\rho_2 = \gamma_2/\gamma_0 = \phi_2$, $\rho_j = \gamma_j/\gamma_0 = \phi_2 \gamma_{j-2}/\gamma_0$

   (e) No spike at $j = 1$ and (positive) spikes slowly decreasing from $j = 2$

3. (a) $\sum_{j=1}^{p} \phi_i < 1$

   (b) $\sum_{j=1}^{p} |\phi_i| < 1$

   (c) $\sum_{j=1}^{p} \phi_i = 1$

   (d) $Y_t = c + 0.1Y_{t-1} + 0Y_{t-2} + \cdots + 0Y_{t-p} + \varepsilon_t$

   (e) $Y_t = c + Y_{t-1} + 0Y_{t-2} + \cdots + 0Y_{t-p} + \varepsilon_t$

4. (a) No, the mean changes over time

   (b) Yes, it appears that the mean and variance are constant

   (c) ARMA

   (d) White noise – the assumptions we assume on the errors appear to be correct

   (e) Forecast