Economics 413: Economic Forecast & Analysis

Department of Economics, Finance and Legal Studies

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Midterm I – Answers

1. (a) No spikes in ACF or PACF (this is a white noise sequence)
   (b) 1 spike in ACF, infinite spikes in PACF
   (c) Infinite spikes in ACF, 1 spike in PACF
   (d) Infinite spikes in ACF (but increasing over time), 1 spike in PACF ($\phi_{11} = 1.4$)
   (e) Infinite spikes in ACF and PACF

2. (a) $\mu = E(Y_t) = E(\varepsilon_t + \theta_2\varepsilon_{t-2}) = 0$
   (b) $\gamma_0 = V(Y_t) = E\left[(Y_t - \mu)^2\right] = E\left(Y_t^2\right) = E\left[(\varepsilon_t + \theta_2\varepsilon_{t-2})^2\right] = E\left(\varepsilon_t^2\right) + \theta_2^2 E\left(\varepsilon_{t-2}^2\right) + 2\theta_2 E(\varepsilon_t\varepsilon_{t-2}) = \left(1 + \theta_2^2\right)\sigma^2$
   (c) $\gamma_1 = E[(Y_t - \mu)(Y_{t-1} - \mu)] = E(Y_tY_{t-1}) = E[(\varepsilon_t + \theta_2\varepsilon_{t-2})(\varepsilon_{t-1} + \theta_2\varepsilon_{t-3})] = 0$; $\gamma_2 = E[(Y_t - \mu)(Y_{t-2} - \mu)] = E(Y_tY_{t-2}) = E[(\varepsilon_t + \theta_2\varepsilon_{t-2})(\varepsilon_{t-2} + \theta_2\varepsilon_{t-4})] = \theta_2\sigma^2$; $\gamma_3 = E[(Y_t - \mu)(Y_{t-3} - \mu)] = E(Y_tY_{t-3}) = E[(\varepsilon_t + \theta_2\varepsilon_{t-2})(\varepsilon_{t-3} + \theta_2\varepsilon_{t-4})] = 0$; and $\forall j > 2$, $\gamma_j = 0$
   (d) $\rho_1 = \gamma_1/\gamma_0 = 0$, $\rho_2 = \gamma_2/\gamma_0 = \theta_2/\left(1 + \theta_2^2\right)$, $\rho_j = \gamma_j/\gamma_0 = 0\ \forall j > 2$
   (e) The ACF will have one *negative* spike at lag 2 ($j = 2$) equal to $\theta_2/\left(1 + \theta_2^2\right)$

3. (a) The ACF measures the correlation between Y_t and Y_{t-j} and the PACF measures the correlation between Y_t and Y_{t-j} controlling for all the periods inbetween. The first spike of each is the sample correlation for the first lag (Y_{t-1}), hence there are no periods inbetween and thus the two correlations will be identical.
   (b) ARMA(0,0) or a white noise process with a mean: $Y_t = \mu + \varepsilon_t$
   (c) An ARMA(0,0) will be $E(Y_t) = \mu$, $\gamma_0 = E\left(\varepsilon_t^2\right) = \sigma^2$ and $\gamma_j = 0\ \forall j$
   (d) ARMA(0,0) has a smaller SC, but everything else points to the ARMA(1,1) model (higher adj R^2, small s.e. of regression and smaller AIC).
   (e) The ARMA(1,1) model can be written as $Y_t = c + \phi Y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$. In 1965 this can be written as $Y_{1965} = c + \phi Y_{1964} + \varepsilon_{1965} + \theta\varepsilon_{1964}$, but we do not have data for 1964. Therefore the earliest period we can start at is 1966. The 1966 model can be written as $Y_{1966} = c + \phi Y_{1965} + \varepsilon_{1966} + \theta\varepsilon_{1965}$ and this is feasible given that we have data in 1965.